LLOYD'S MIRROR—IMAGE INTERFERENCE EFFECTS

William M. Carey
Department of Mechanical Engineering, Boston University
Boston, Massachusetts 02215

Introduction

On a calm day, an underwater source of sound has its radiation reflected from the surface of the sea much as if it were a mirror with an "out-of-phase" image. At moderate distances from the source of sound an underwater receiver would detect two signals—an acoustic pressure fluctuation due to the direct arrival of sound from the source and a second signal from its reflected mirror image. If the arrivals combine constructively (in-phase) a loud sound is observed and likewise when the arrivals combine destructively (out-of-phase) a fade is observed. Successive regions of loudness and fading are referred to as an *image interference* effect. The combination of a source near an out-of-phase reflecting surface is referred to as *Lloyd's Mirror.*

"An excellent example of the importance of experimental work based on theory for the production of clear unambiguous results."

The image interference effect has been central to investigations in underwater acoustics and sonar throughout the last sixty years and has been simply referred to as the Lloyd's Mirror effect. It was so well known that even the "Red Books," (*The Physics of Sound in the Sea*), did not provide a reference to Lloyd. Research during World War II was conducted by the National Defense Research Committee (NDRC) that was established by Vannevar Bush under the authority of a Presidential Executive Order in 1940. The Committee later became a part of the Office of Scientific Research and Development. In 1945, the NRDC issued a twenty volume technical report summarizing this research and the "Red Books," of high interest to underwater acousticians, were the summary volumes of Division 6: Sub-Surface Warfare. These volumes were issued by Tate (1945, 1946) and reprinted by the Navy Material Command (NAVMAT) in 1969. They comingled the significant work of many individuals and institutions, Table 1.

If identified, the Division 6 principal investigators would be recognized as the founders of underwater acoustics and signal processing that are the underpinnings of much of our current-day endeavors. However, their individual contributions during this short period of time were unselfishly shared. Although it is difficult to be specific as to the individual contributions, nevertheless, it is beyond question that the work of the assembled scientists and engineers presented in this five year time frame was both remarkable and significant. The legacy of this war-time research is still reflected in the present-day structure of applied acoustic research and development laboratories.

Table 1. National Defense Research Council (NRDC): Division 6: Sub-surface Warfare

- University of California Division of War Research w/USN Radio and Sound Laboratory
- Woods Hole Oceanographic Institution
- USN Sound Reference Laboratories w/Columbia University Division of War Research New London Laboratory
- Massachusetts Institute of Technology Underwater Sound Laboratory

The interference problem was referred to by these wartime researchers, as the image interference effect, the Lloyd Mirror Effect. With the addition of an effective surface reflection coefficient the method of images was found to be useful in sound transmission experiments using both continuous and impulsive sources of sound and in sonar applications such as multi-path ranging. This article will discuss first, the scientific, technical and practical aspects of the effects. Following, will be a discussion of Lloyd, the scientist and mathematician, that history seemed to have forgotten.

The Lloyd's Mirror Effect—Image interference effect

Investigators were first concerned with the signals received at distance from explosives used to determine sound transmission characteristics. The signals received consisted of a sequence of a direct signal followed by a negative surface-reflected arrival. This group was called a poke. The reflection from the sea surface was described by an image with a change of sign and the relative amplitude of the arrivals was accounted for by use of an effective reflection coefficient. Subsequent experiments with gated sine wave signals were described by this same method of images with the effective reflection coefficient for sonar frequencies (< 10 kHz) based on sea state. Young (1947) and Urick (1967) extended the treatment to include refraction, broad band signals and a realistic treatment of the surface reflection coefficient.

The basic approach and results of this image interference effect can be understood by the superposition of a source with strength, ρ_{os}, beneath the pressure release surface and its image strength multiplied by an effective coefficient, $\mu\,\rho_{os}$. See Fig. 1. Since pressures are additive, the received pressure at R is the sum of the radiated pressure from the source and its image:

$$p[r,t] = p_s[r_s,t] + p_i[r_i,t]. \qquad (1)$$

The radial distances from the source and its image in the

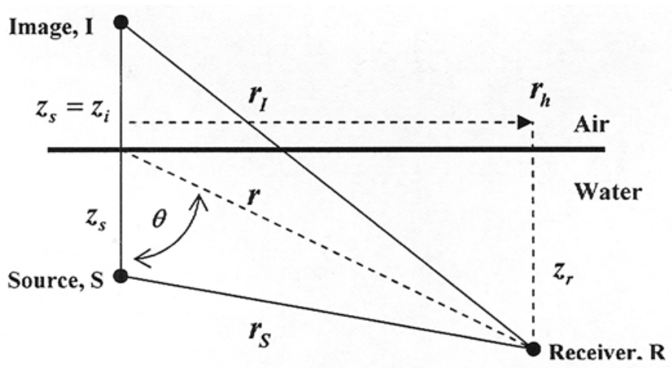

Fig. 1. The Lloyd Mirror geometry.

absence of refraction can be determined directly from the geometry

$$r_{s,i} = [r_h^2 + (z_r \mp z_{s,i})^2]^{1/2}, \ |z_s| = |z_i|. \qquad (2)$$

When one assumes simple harmonic sources, exp (-$i\omega t$), with outward propagating spherical waves, exp (-ikr), the resultant pressure at the receiver R, ρ(r_h, z_r, t), with a surface reflection coefficient μ is

$$p[r,t] = [p_{os} / r_s] \times \exp[ikr_s - i\omega t] + [\mu p_{os} / r_i] \times \exp[ikr_i - i\omega t]\,. \qquad (3)$$

One can define three regions: a nearfield, an interference field and a farfield. The nearfield is when the direct arrival from the source is dominant. The nearfield region is defined, following Urick (1967), as distances less than the range, r_{hnf}, at which the intensity of the image source is ½ that of the intensity of the direct source arrival. Expanding the source and image radial distances, in Eq. 3, with a Taylor series and neglecting the second order terms, $z^2_{s,i}/r^2$, yields the expression for intensity at R:

$$I = (1/2\rho c)\mathrm{Re}(pp^*) = (p_{os}^2 / 2\rho c) \cdot [1/r^2] \cdot [1 + \mu^2 + 2\mu \cos(2kz_r z_s / r)] \qquad (4)$$

When the surface reflection coefficient, μ, is equal to (-1) the resultant intensity is proportional to two times the source intensity times the bracketed cosine term.

$$I = I_{os}[1/r^2] \cdot 2 \cdot [1 - \cos(2kz_r z_s / r)]. \qquad (5)$$

The argument of the cosine determines the maxima and minima in the intensity as a function of r. Maxima occur when the cosine is equal to -1 for

$$r_{\max,n} = 4z_r z_s / (2n+1)\lambda, \ n = 0,1,2,\ldots \ \text{and} \ I = 4I_{os} / r^2. \qquad (6)$$

The interference peaks at these ranges are four times the free field intensity, I_{os}.

The above specify the interference field and the farfield expression can be obtained by use of trigonometric relationships

$$I = I_{os}[1/r^2] \cdot [1 + \mu^2 + 2\mu(1 - 2\sin(kz_r z_s / r)^2] \quad \text{and with } (\mu = -1) \rightarrow I_{os}[1/r^2] \cdot [4\sin(kz_r z_s / r)^2]. \qquad (7)$$

First at a constant distance, r, the quantity

$$z_r / r = \cos(\theta); \ \ I(r) r^2 / I_{os} \approx 4(kz_s)^2 \cos(\theta)^2; \qquad (8)$$

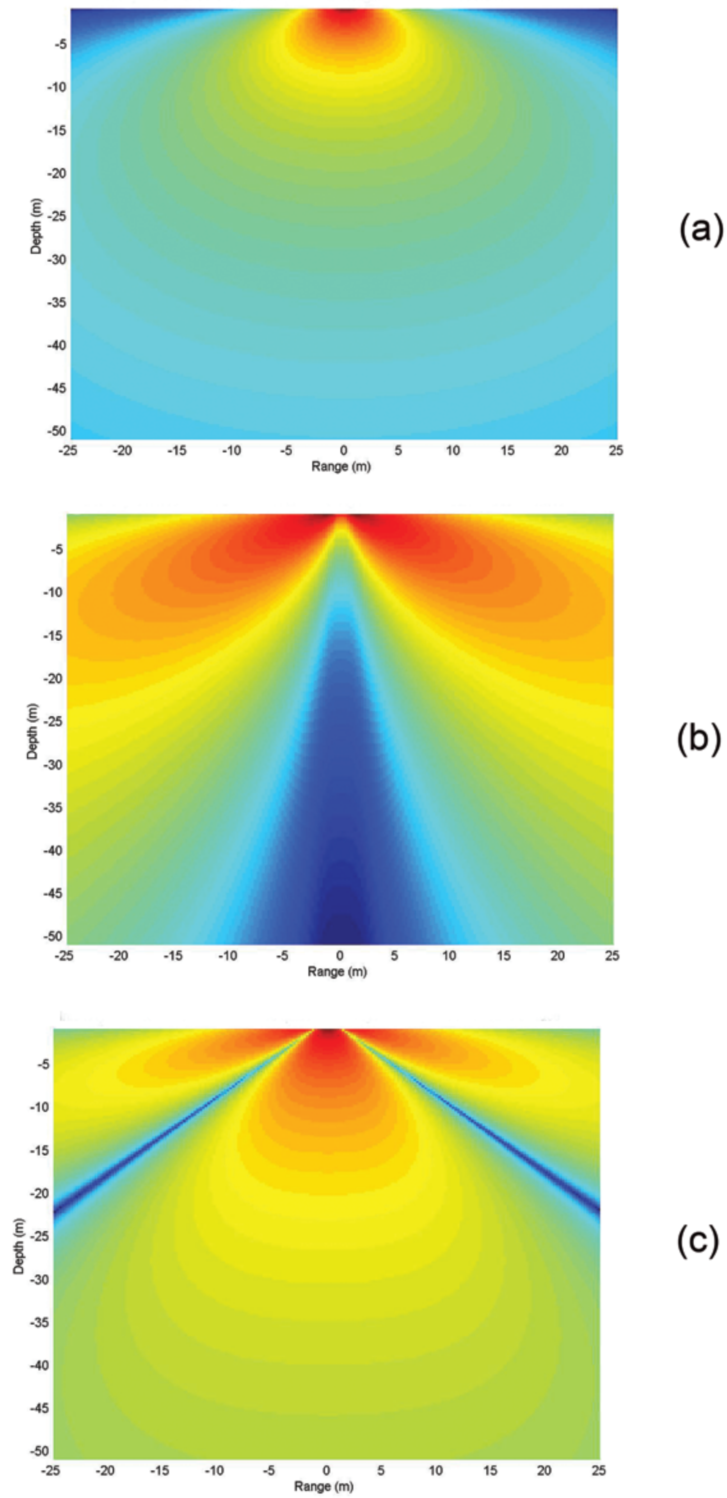

Fig. 2. The change in the vertical directionality of the monopole source beneath the pressure release surface, (a) monopole source λ/4 below the pressure release surface, (b) monopole source λ/2 below the pressure release surface, (c) monopole source 3λ/4 below the pressure release surface.

The directional radiation pattern is a *dipole* pattern and the amplitude depends on $(2\pi z_s/\lambda)$ the proximity of the source to the pressure release surface. As the source approaches the surface, $z_s \rightarrow 0$, it collides with its image and the result is zero, the characteristic of a doublet.

The mean-square pressure in the far field is

$$|p(r)|^2 / p_{os}^2 = \\ 4(2\pi z_s / \lambda r)^2 \cos(\theta)^2 = 4(2\pi z_s z_r / \lambda)^2 / r^4. \quad (9)$$

The farfield mean-square pressure decreases with the radial distance to the fourth power. This doublet characteristic is a consequence of the monopole beneath a pressure release surface. On the other hand the mathematical "point" dipole is derived by placing two monopoles of opposite sign separated by a distance $2\Delta z_s$ and taking the limit as $\Delta z_s \rightarrow 0$ and $2p_{os}\Delta z_s \rightarrow D$ the dipole source strength:

$$p_D = ikD\cos(\theta)(1 + i / kr)\exp(ikr). \quad (10)$$

The subtle but pertinent issue is that a bubble below a pressure release surface has on average a dipole characteristic referred to here as a doublet; however as $z_s \rightarrow 0$ the radiated pressure goes to zero. On the other hand, the point dipole, such as a rain drop impact on the pressure release surface, radiates sound with dipole strength, D, and the following characteristic

$$|p_D|^2 = k^2 D^2 \cos(\theta)^2 (1 + 1 / k^2 r^2) / r^2; \quad (11)$$

the reactive term, $1/k^2r^2$, becomes negligible at reasonable distance from the source.

The difference between the point dipole and the doublet is fundamental. For near surface sources one should expect a *dipole* radiation pattern as shown in Fig. 2a. However as the depth of the source increases the pattern becomes more complex as shown in Figs. 2b and 2c. This effect can be observed with submerged sources such as large surface ship propellers that are generally at depths of less than a half of a wave length at shaft and blade-rate frequencies. Sound radiation from these propellers is also influenced by the hull in the forward direction and the wake in the aft direction. This results in a horizontal radiation pattern which is also cosinusoidal.

In the frequency range between 100 Hz and 1 kHz, the image-interference pattern can be observed at considerable horizontal distances. These transmission characteristics are shown in Fig. 3. where the relative level, RL, is plotted versus range illustrating the near field, interference field and the far field, the Lloyd's mirror range.

$$RL = 10\log(Ir^2 / I_{os}) = \\ 10\log(4\sin(kz_r z_s / r)^2) = -TL + 20\log(r) \quad (12)$$

The discussion to this point has simply dealt with the case where the reflection coefficient was unity. If the sea state spectral density is written in terms of the roughness parameter h; the intensity can be shown proportional to $\mu_o \approx$-1 and the acoustic roughness $R=2khsin\theta_g$, the Rayleigh parameter.

$$< p_r >^2 / p_i = I_r / I_i = \mu_o^2 \cdot \exp[-(2kh\sin\theta_g)^2] \quad (13)$$

This formulation can be useful in determining the effect on the effective reflection coefficient μ. In the mid frequency range the increase in μ fills in the nulls of the interference pattern and reduces the magnitude of the peaks. The interesting feature is that for low sea states, Beaufort Number 3 (wind speeds between 3.4–5.4 m/s) the ratio of the reflected to incident intensities is less than approximately 0.86 for grazing angles less than thirty degrees. Examples of at-sea measurements performed in the 1980's at low sea states can be found in Carey (1997), see Figs. 4 and 5.

The utility of this image interference pattern is seen in the at-sea calibration of array hydrophone groups shown in Fig. 5. Common practice used in construction of seismic-type arrays was to combine multiple hydrophones connected in series and parallel to form groups with physical lengths on the order of a quarter wave length. Even though individual calibrations were usually performed on each hydrophone, the calibration of the hundreds of array hydrophone groups with the long transmission cable was desirable. Such calibration geometry is shown in Fig. 4. This technique requires stable and consistent motion of the vessel, good ship driv-

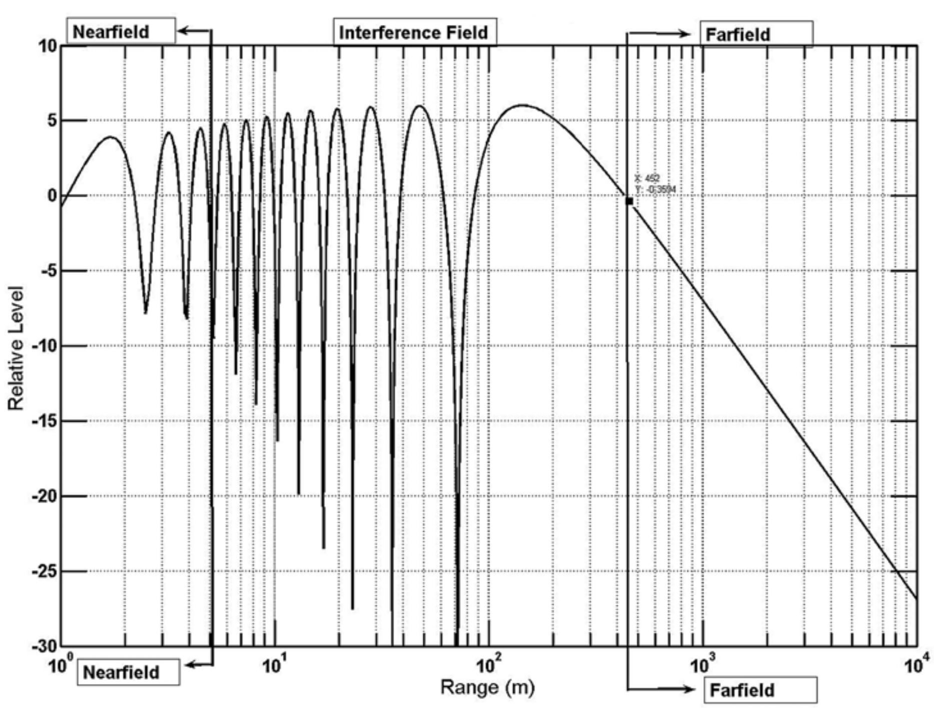

Fig. 3. The mid-frequency, 5 kHz, interference pattern is shown versus range for a reflection coefficient (μ= -0.9), a source depth of 6λ, and a receiver depth of 20 λ.

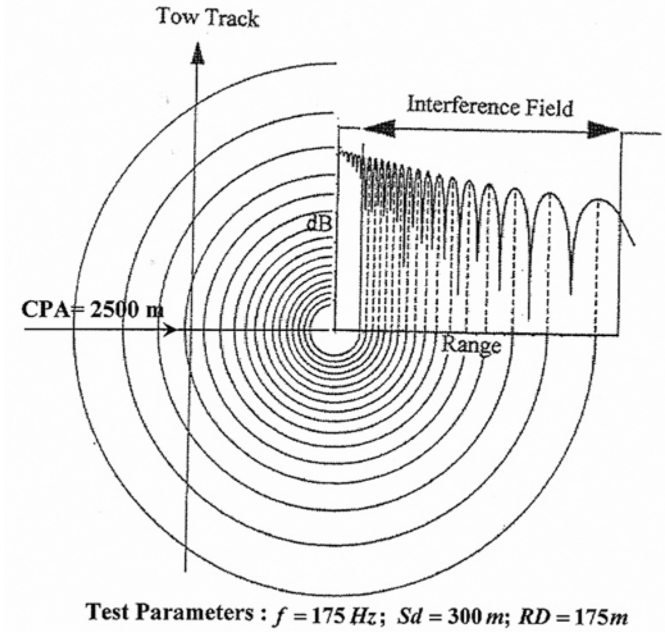

Fig. 4. Planar view of tow geometry for calibration of a low frequency hydrophone group. In this example the ship and array proceed in a constant speed and heading and consequently a constant array depth. The interference pattern is determined by the closest point of approach and is characteristic for a constant source receiver depth.

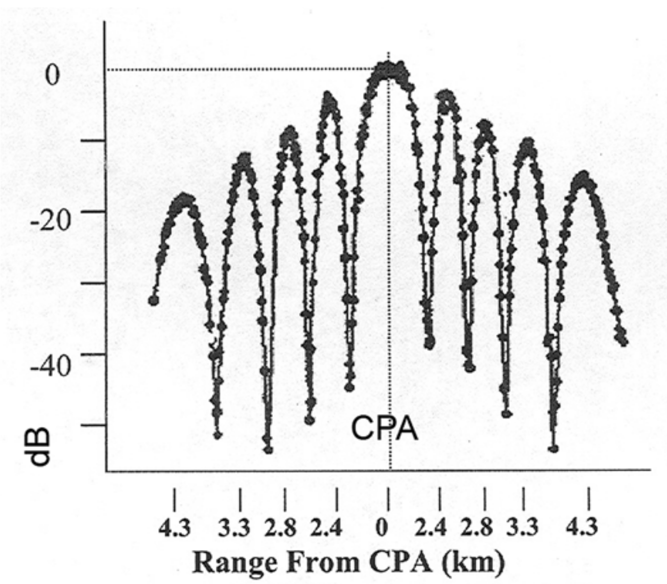

Fig. 5. Measurements of the Lloyd Mirror effect for Beaufort Sea State 3 with an array composed of hydrophone groups of 20 series-parallel connected hydrophones. The array was towed at a nominal depth of 175 m past a moored acoustic source at a depth of 300 m radiating a tone at 175 Hz. The closest point of approach, CPA, was 2500 m.

ing, and a balanced array in a stable tow configuration. The corresponding results for this experiment conducted at constant ship rpm for a speed of 2.9 knots with the array at 175 m depth, a moored sound source radiating sound at 175 Hz at 300 m depth with the closest point of approach of 2500 m, are shown in Fig. 5. This particular array had 105 hydrophones and the remarkably consistent results are attributable to stable oceanographic and sea state conditions. An interesting feature of this calibration is the use of the last interference peak to calibrate the beam forming in the end fire direction. This technique was used prior to several major sea tests and the results were successful.

This image interference effect may also be important in determining the response of near surface marine mammals to shipping noise. Is this effect the reason dolphins approach ships from an angle to ride the bow wave impervious to the radiated noise? Is this the reason that near surface whales can not hear an approaching tanker? What role does this effect have on the increased noise levels and, even though tonnage and power is increasing, does the change in propeller efficiency and depth offset these increases? These questions are of current interest and certainly the image interference effect will be an important factor even though sophisticated range and frequency dependence are used to describe the environmental impact of industrial noise.

Thus the image interference effect—the Lloyd's Mirror Effect—has continually been an important factor in underwater acoustics since World War II. All texts on underwater and marine acoustics, in my library cite this effect, but few if any provide a reference to Lloyd. Who was Lloyd and why is he so widely cited in optics and acoustics but rarely referenced? Perhaps this was the result of a common practice of only citing previous texts and not primary references.

Humphrey Bartholomew Lloyd—the scientist and mathematician

Initially the thought was that Lloyd's identity would be a simple question to answer—using library resourses and rummaging in the stacks. However there are no longer card indexes or stacks. Simply put, there are only digital searches and warehouses. Initial searches using the library search engine and Google were not productive. However discovery of the Cantor (1983) text, opened doors. With Cantor's clue to whom Lloyd was, the power of our digital universe was unlocked and the questions, "Who was Lloyd and why is he cited?" were answered.

In 1812, Bartholomew Lloyd, Chair of Mathematics, used methods based on the calculus of Liebniz and the work of Laplace and Lagrange from his studies at the Ecole Polytechnique in Paris [Cantor (1983, 149)] to restructure the mathematics curriculum of Trinity College, Dublin. His efforts were successful in producing several excellent mathematicians such as his son Humphrey Lloyd (Fig. 6, Table 2) and the renowned William Hamilton (of the Hamiltonian fame). The fundamental importance of this endeavor was that it resulted in the recognition of the importance of mathematical potential as a guide to the physical analysis of phenomena such as optics. In short, experiments, either based on theoretical expectations or designed to test theory, were introduced to mid-19th century optical science.

Although Humphrey Lloyd was acclaimed for his 1831 Treatise on Light and Vision published in London and unavailable to this author, he apparently had an improved and more wave theoretic interpretation of plane optics by 1834. (Note, that in the 19th century it was the date when the paper was read before the Royal Societies rather than the date of publication in the Transaction or Proceedings). Humphrey Lloyd's most remarkable single scientific achievement occurred in 1832, namely, the experimental proof of the phenomenon of conical refraction, the production of a luminous cone of light by a crystal, predicted by the mathematical extension of the theory of Fresnel by Hamilton. This discovery was a rare instance in which theory was not only able to mathematically describe the phenomenon but to predict it. The experiment had been suggested by Hamilton to test his theoretical predication of conical refraction. As stated in the Royal Science Obituary, 1881, "It would be impossible to give here a detailed account of the difficulties attendant upon this inquiry. Suffice it to say that they were overcome by the experimental ability of Dr. Lloyd, who succeeded in giv-

Fig. 6. Humphrey Lloyd (1800-1881), Provost of Trinity College Dublin, Clergyman of the State Church of Ireland and member of the Protestant Ascendancy, the British ruling class of Ireland.

Table 2. Humphrey Bartholomew Lloyd (1800–1881)

Born: Dublin, Ireland (British)
Profession: Clergyman
Research: Natural Philosophy, optics, terrestrial magnetism
Science Gold Medal (1818)
Trinity College: BA (1819); MA (1827); DD (1840)
Oxford: DCL, Honoris Causa (1855)
Fellow ('24)-Professor ('31)-Provost ('67); Trinity College
Fellow of the Royal Society (1836)
Fellow and president, Royal Irish Academy (1846-51)
President of the British Association in Dublin (-1857)
German Cross of the Order "Pour le Mérite" (1874)
Cunningham Medal of Royal Irish Academy (1862)

Table 3. Key image interference experiments

Young's Double-slit Experiment (1807)
Fresnel's Biprism (1819)
Fresnel's Double Mirror Experiment (1819)
Lloyd's Conical Refraction Experiment (1833)
Lloyd's Mirror Experiment (1834)
Billet's Split Lens (1858)

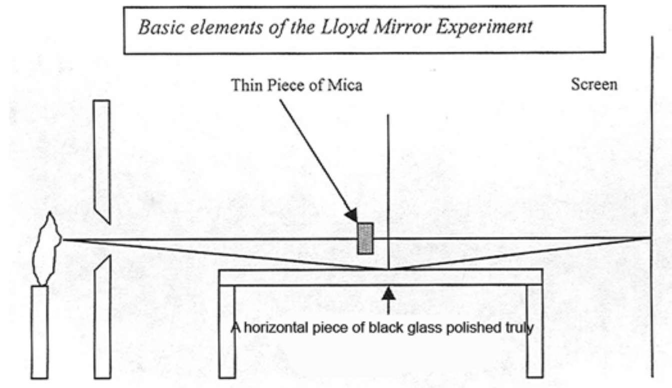

Fig. 7. The Lloyd Mirror Experiment schematic based on the text of Lloyd (1837) and the work of Mach (1926).

ing a perfect experimental demonstration of this remarkable phenomenon in both its varieties." Young (1807) and Fresnel (1819) (see Table 3) had previously performed experiments to explain this refraction phenomena but the experiments performed between 1831 and 1833 by Lloyd were characterized by such experimental skill and clarity as to set them apart.

The conical refraction experiment definitively verified the predictions of Hamilton and was perhaps the earliest combination theory and measurement to describe the wave nature of light. The results of the conical refraction experiment were considered important as there was still considerable debate as to whether projectile, fluid, vibration or wave theory was the correct theoretical description of optical interference effects Cantor (1983). He prepared a progress report on optical science for the British Association between 1833 and 1834. Shortly after, in 1834, he performed the mirror experiment shown in Fig. 7. And described his results to the Royal Irish Academy.

The importance of the elegant design and clarity of the result along with the wave theoretic explanation were immediately recognized. In fact, his analytical explanation is similar to that described here and found in *Physics of Sound in the Sea*. Lloyd recognized the importance of the phase of his reflection from his polished glass and concluded that a phase change of π resulted. He also recognized that the introduction of a phase shift on the direct wave, the piece of mica, that would shift the pattern so that the complete interference pattern could be observed. His estimation of the peak intensities and observations of the nulls were clearly explained in his paper read in 1834 and published in the Transactions in 1837. Mach (1926) cites a later experiment by Billet (Table 3) that produced comparable results; but it was the experiment by Lloyd that was considered most seminal. These early contributions to optics were probably the reason he became a Fellow of the Royal Society in 1836. Selected optical references to Lloyd are appended for those interested in his texts and original works.

Between 1831 and 1833 the British Association decided to conduct surveys of the terrestrial magnetism field intensity in the kingdom. A standing committee, including Lloyd was formed and he undertook to make the required observations in Ireland with Captain Sabine. He developed, along

Table 4. Major contributions

A Treatise on Light and Vision (1831)
Conical Refraction through Bi-axial Crystals (1831-1833)
Lloyd's Mirror Experiment (1834)
With Sabine measurement Instrument for magnetic fields (1834)
Establishment: School of Eng. at Trinity College (1841)

with Captain E. Sabine FRS, an instrument that accurately measured the dip and intensity of the magnetic field by observation of a magnet turning round a horizontal axis. He and Sabine made important magnetic field observations in Ireland and was a significant contributor to the British Association's Terrestrial Magnetism program that resulted in the establishment of many observatories to study the temporal changes in the Earth's magnetic field in Britain and later, the world. Dr. Lloyd was responsible for the methods used in these observatories and analysis of the global results. During this period of time he contributed to the establishment in 1841 of the School of Engineering at Trinity College, Dublin, see Table 4.

Humphrey Lloyd was considered the most distinguished scholar to hold the position of Provost at Trinity College, Dublin, since N. Marsh (1679). He was honored by Oxford with a Doctorate Honoris Causa (1855), with the Cunningham medal of the Royal Irish Academy (1862), and with the German Cross of the Order "Pour le Mérite" (1874). Lloyd was ahead of his time with his recognition of the importance of university research. The image interference effect and the investigations of Lloyd are indeed an excellent example of the importance of experimental work based on theory for the production of clear unambiguous results. Perhaps this is the real significance of the 19th century Lloyd and explains his acclaim.**AT**

References

Cantor, G. N. (**1983**). *Optics after Newton, Theories of Light in Britain and Ireland*, 1704–1840 (Manchester University Press, Manchester, UK).

Carey, W., Reese, J., and Stuart, C. (**1997**). "Mid-frequency Measurements of Array Signal and Noise Characteristics," J. Oceanic Engineering **22**(3), 548–565.

Jenkins, Francis, and White, Harvey (**1950**). *Fundamentals of Optics 2nd edition* (McGraw-Hill Book Company Inc., New York) pp. 225–239.

Mach, Ernst (**1926**). *The Principles of Physical Optics* (Dover Publications, Inc, New York) pp. 164–168.

Tate, J. T. (**1945**). *Physics of Sound in the Sea*, edited by L. Spitzer (First published 1945 by National Defense Research Council, Reprinted 1969 by Naval Material Command, LOC 88-06268, ISBN 0-9321-24-4). Republished by Peninsula Publishing, Los Altos, CA.

Tate, J. T. (**1946**). *Principles and Applications of Underwater Sound*, edited by C. Eckart, Dept. of Navy Headquarters of the Naval Material Command, Washington, DC. Available from GPO,1968-{Originally issued as the Summary Technical Report of Division 6, NDRC, Vol. 7, 1946) pp. 24-25, pp. 44–48.

Urick, R. (**1967**). *Principles of Underwater Sound for Engineers*, pp 110–114. (also see 3rd edition, Peninsula Publishing, Los Altos, CA, 1983, pp.131–134,).

Young, R. W. (**1947**). "Image interference in the presence of refraction," J. Acoust. Soc. Am. **19**(1), 1–7.

Humphrey Lloyd References

Lloyd, Humphrey (**1831**). *A Treatise on Light and Vision.*

Lloyd, Humphrey (read **1834**). "On a New Case of Interference of the Rays of Light," Transactions of the Royal Irish Academy **17**, 171–177 (1837).

Lloyd, Humphrey (**1873**). *Elementary Treatise on the Wave Theory of Light, 3rd Ed*. (Longmans, Green, and Co., London (Available Google Books).

Royal Society Obituary (**1881**). Proc. Royal Society, 1880–1881 (31) pp. xxi-xxvi, signed JHJ

Lloyd, Humphrey (**1877**). *Miscellaneous Papers Connected With Physical Science* (Longmans, Green, and Co., London (Available Google Books)
Contains:

Lloyd, Humphrey (**1833**). "On the Phenomena Presented by Light in its Passage Along the Axes of Biaxial Crystals," Philosophical Magazine **2**, 112–120, 207–210 (1833).

Lloyd, Humphrey (**1837**). "On a New Case of Interference of the Rays of Light," Read January 27, 1834, Trans. Royal Irish Academy XVII (Royal Irish Academy of Sciences, Dublin).

William (Bill) Carey received his education in engineering and physics at The Catholic University of America. He is an Associate Editor for Signal Processing, the *Journal of the Acoustical Society of America*. Bill is also an Associate Editor (Formerly Chief Editor) of the *Journal of Oceanic Engineering*. Currently he is a Professor of Mechanical Engineering at Boston University, an Adjunct Professor of Applied Mathematics at the Rensselaer Polytechnic Institute, and an Adjunct Scientist at the Woods Hole Oceanographic Institution. Previously, he was a Physicist with the Advanced Research Projects Agency assigned to the Massachusetts Institute of Technology Department of Ocean Engineering, where he taught graduate courses in Acoustics. Bill was a Research Physicist and Engineer affiliated with the Navy's Research and Development Laboratories. At the University of Chicago's Argonne National Laboratory, he was responsible for acoustic surveillance of power plants. He has been a consultant to both industry and government in the areas of nondestructive testing, nuclear science-environmental measurements, and applied ocean acoustics. Bill Carey, a Fellow of the Society, recently received the Pioneers of Underwater Acoustics Medal. Bill is a Fellow of the Oceanic Engineering Society and has received the IEEE Oceanic Engineering Society's Distinguished Technical Achievement Award, Distinguished Service Award, and an IEEE Millennium Award.