THE PARADOX OF PITCH CIRCULARITY

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Introduction

In viewing M. C. Escher's lithograph *Ascending and Descending* shown on the front cover, we see monks plodding up and down an endless staircase—each monk will ultimately arrive at the place where he began his impossible journey. Our perceptual system insists on this interpretation, even though we know it to be incorrect. The lithograph was inspired by the endless staircase devised by Lionel Penrose and his son Roger Penrose,[1] a variant of which is shown in Fig. 1. Our visual system opts for a simple interpretation based on local relationships within the figure, rather than choosing a complex, yet correct, interpretation that takes the entire figure into account. We observe that each stair that is one step clockwise from its neighbor is also one step downward, and so we perceive the staircase as eternally descending. In principle, we could instead perceive the figure correctly as depicting four sets of stairs that are discontinuous, and viewed from a unique perspective—however such a percept never occurs.

"The phenomenon of pitch circularity has implications for our understanding of pitch perception, as well as for musical composition and performance."

This paper explores an analogous set of auditory figures that are composed of patterns that appear to ascend or descend endlessly in pitch. Here also, our perceptual system opts for impossible but simple interpretations, based on our perception of local motion in a particular direction–either upward or downward. These sound patterns are not mere curiosities; rather they provide important information concerning general characteristics of pitch perception.

Pitch as a two-dimensional attribute

By analogy with real-world staircases, pitch is often viewed as extending along a one-dimensional continuum of *pitch height*. For sine waves, any significant increase or decrease in frequency is indeed associated with a corresponding increase or decrease in pitch—this is consistent with a one-dimensional representation. However, musicians have long acknowledged that pitch also has a circular dimension, known as *pitch class*—tones that stand in octave relation have a certain perceptual equivalence. The system of notation for the Western musical scale accommodates this circular dimension. Here a note is designated by a letter which refers to its position within the octave, followed by a number which refers to the octave in which the tone occurs. So as we ascend the scale in semitone steps, we repeatedly traverse the pitch class circle in clockwise direction, so that we play C, C#, D, and so on around the circle, until we reach C again, but now the note is an octave higher. Similar schemes are used in Indian musical notation, and in those of other musical cultures.

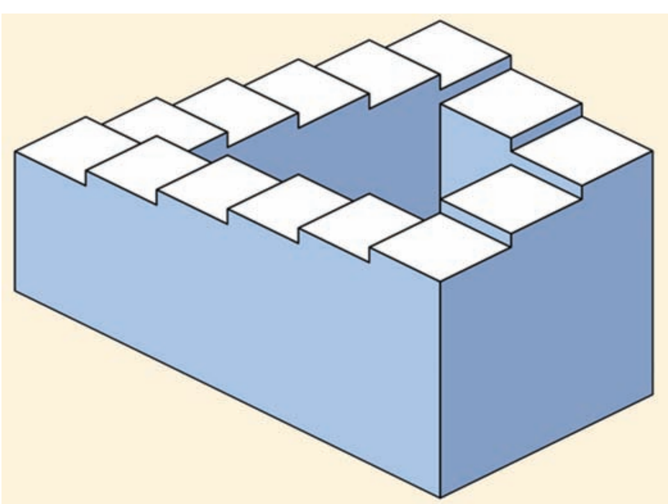

Fig. 1. An impossible staircase, similar to one devised by Penrose and Penrose.[1]

To accommodate both the rectilinear and circular dimensions of pitch, a number of theorists—going back at least to Drobisch in the mid-nineteenth century—have argued that this be represented as a helix having one complete turn per octave, so that pairs of points that are separated by an octave stand in close spatial proximity (Fig. 2). Based on such a representation, Roger Shepard, then at Bell Telephone Laboratories, conjectured that it might be possible to exaggerate the dimension of pitch class and minimize the dimension of height, so that all tones that are related by octaves would be mapped onto a single tone which would have a well-defined pitch class but an indeterminate height. Because the helix would then be collapsed into a circle, judgments of relative pitch for such tones should be completely circular.[2, 3]

Using a software program for music synthesis generated by Max Mathews,[4] Shepard synthesized a bank of complex tones, each of which consisted of 10 partials that were separated by octaves. The amplitudes of the partials were scaled by a fixed, bell-shaped spectral envelope, so that those in the middle of the musical range were highest, while the amplitudes of the others fell off gradually along either side of the log frequency continuum, sinking below the threshold of audibility at the extremes (Fig. 3). Such tones are well defined in terms of pitch class (C, C#, D; and so on) but poorly defined in terms of height, since the other harmonics that would provide the usual cues for height attribution are missing. Using such a bank of tones, one can then vary the dimensions of height and pitch class independently. To vary height alone one can keep the partials constant but rigidly shift the spectral envelope up or down in log frequency; to vary pitch class alone one can rigidly shift the partials in log frequency, while keeping the position of the spectral envelope constant.

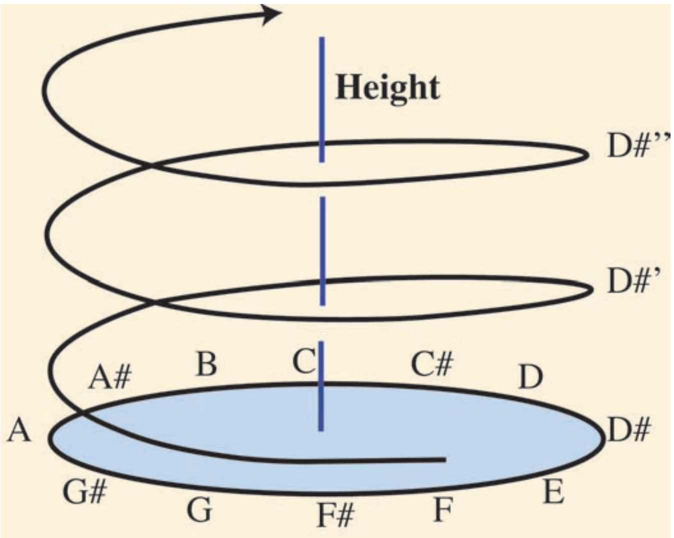

Fig. 2. The helical model of pitch. Musical pitch is depicted as varying along both a linear dimension of height and also a circular dimension of pitch class. The helix completes one full turn per octave, so that tones that stand in octave relation are in close spatial proximity, as shown by D#, D#', and D#."

To demonstrate that such tones have circular properties when the position of the spectral envelope remains fixed, Shepard presented listeners with ordered pairs of such tones, and they judged for each pair whether it formed an ascending or a descending pattern. When the tones within a pair were separated by a small distance along the pitch class circle, judgments of relative height were determined entirely by proximity. As the distance between the tones increased, the tendency to follow by proximity lessened, and when the tones were separated by a half-octave, averaging across pitch classes and across a large group of subjects, ascending and descending judgments occurred equally often.[2]

Shepard then employed such a bank of tones to produce an intriguing demonstration: When the pitch class circle is repeatedly traversed in clockwise steps, one obtains the impression of a scale that ascends endlessly in pitch: C# sounds higher than C; D as higher than C#, D# as higher than D; A# as higher than A; B as higher than A#; C as higher than B; and so on endlessly. One such scale is presented in **Sound Demonstration 1** (See Sidebar). When the circle is traversed in counterclockwise steps, the scale appears to descend endlessly instead. This pitch paradox has been used to accompany numerous videos of bouncing balls, stick men, and other objects traversing the Penrose staircase, with each step accompanied by a step along the Shepard scale.

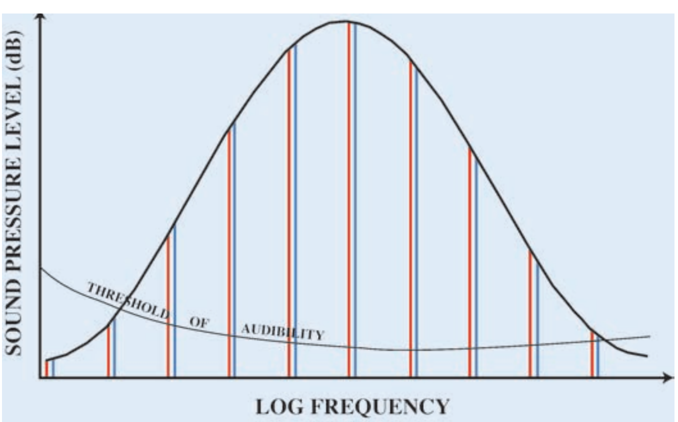

Fig. 3. Spectral representation of Shepard's algorithm for generating pitch circularity. Circularity is obtained by rigidly shifting the partials up or down in log frequency, while the spectral envelope is held fixed. As examples, the red lines represent partials at note C, and the blue lines represent partials at note C#. Adapted from Shepard.[2]

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Jean-Claude Risset has produced remarkable variants of this illusion, using the same basic principle.[5,6] One set of variants consists of endlessly ascending and descending glissandi. **Sound Demonstration 2** presents an example. In other variants, the dimensions of pitch height and pitch class are decoupled by moving the position of the spectral envelope in the direction opposite that of movement along the pitch class circle.[5, 6] For example, the spectral envelope could be continuously rising, while the tones traverse the pitch class circle in counter-clockwise direction, so that the listener perceives a sequence that both ascends and descends at the same time. Risset has incorporated many of such glides into his compositions, with striking artistic effect. For example, he employed an endlessly descending glissando in the incidental music to Pierre Halet's Little Boy. This play portrays the nightmare of a pilot who took part in the destruction of Hiroshima, and the descending glide symbolizes the falling of the atomic bomb.

Circularities based on spectral proximity

For Shepard tones, there are two ways to interpret the perceptual tendency to form relationships based on pitch proximity. One possibility is that we invoke proximity along the pitch class circle, which is illustrated in Fig. 2. Another possibility is that we connect together the individual partials of the successive tones based on their proximity along the frequency continuum, as illustrated in Fig. 3. That spectral factors alone can produce circularity effects was demonstrated by Jean-Claude Risset[5, 6] when he produced endlessly ascending and descending glissandi consisting of tone complexes whose partials stood in ratios other than an octave. An experimental demonstration of this spectral proximity effect was later produced by Edward Burns, who created banks of tones whose partials were separated by various ratios, ranging from 6 to 16 semitones. He found essentially no difference in circularity judgments depending on whether octave ratios were involved.[7] Other research demonstrating the contribution of spectral proximity to pitch circularity has been carried out by Ryunen Teranishi,[8] and by Yoshitaka Nakajima and his colleagues.[9]

Pitch circularities in musical practice

Although stark pitch circularities were not created until exact control of acoustic parameters became possible in the mid-twentieth century, overall impressions of pitch circularity have been generated by composers from the Renaissance onward.[10] English keyboard music of the sixteenth century, such as composed by Orlando Gibbons, included clever manipulations of tone sequences in multiple octaves so as to create circular effects. In the eighteenth century, J. S. Bach was strikingly effective in devising passages that gave circular impressions, most famously in his organ *Prelude and Fugue in E minor*.

In the early twentieth century, Alban Berg produced an effect that approached that of circularity generated by Shepard tones. In his 1925 opera Wozzeck, Berg employed a continuously rising scale that was orchestrated in such a way that the upper instruments faded out at the top of their range while the lower instruments faded in at the low end. Other twentieth century composers such as Bela Bartok and Gyorgy Ligeti orchestrated sequences that gave rise to circular impressions. In particular, Jean-Claude Risset has made extensive use of circular configurations in his orchestral works; for example in his piece *Phases* he orchestrated circular configurations using harps, celesta, strings, percussion, and brass.

Twentieth century electroacoustic music has also employed circular effects. These occur, for example, in Risset's *Mutations 1*; James Tenny's *For Ann (rising)*, Karlheinz Stockhousen's *Hymnen*, and the Beatle's *A Day in the Life (Sergeant Pepper)*. Recently, Richard King, sound designer for the Batman movie *The Dark Knight*, employed an ever-ascending glide for the sound of Batman's vehicle, the Batpod. Explaining his use of this sound in the Los Angeles Times, King wrote: "*When played on a keyboard, it gives the illusion of greater and greater speed; the pod appears unstoppable.*"[11]

Towards circular banks of musical instrument tones

To achieve pitch circularity, must our choice of musical material be confined to highly artificial tones, or to several instrument tones playing simultaneously? Alternatively, might it be possible to create circular scales from sequences of single tones, with each tone comprising a full harmonic series? If this could be achieved, then the theoretical implications of pitch circularity would be broadened. Furthermore, this would open the door to creating circular banks of tones derived from natural instruments, which would expand the scope of musical materials available to composers and performers.

Arthur Benade stated that a good flautist, while playing a sustained note, can smoothly vary the amplitudes of the odd numbered harmonics relative to the even-numbered ones, so as to produce an interesting effect.[12] Suppose he begins with a note at ***F***0 = 440 Hz; the listener hears this as Concert A, well defined in both pitch class and pitch height. If the flautist then alters his manner of blowing so as to progressively reduce the amplitudes of the odd harmonics relative to the even ones, the listener will at some point realize that he is no longer hearing Concert A, but rather the A an octave higher (corresponding to ***F***0 = 880 Hz). Yet the transition from the lower to the higher octave can appear quite smooth. Based on this observation, one can surmise further that a tone consisting of a full harmonic series might be made to vary continuously in height between octaves without necessarily traversing the path specified by the helical model, but rather by traversing a straight path upwards or downwards in height—for example between D# and D#' in Fig. 2. Pitch might then be represented as a solid cylinder rather than a helix. **Sound Demonstration 3** presents a harmonic complex tone with ***F***0 = 440 Hz, in which the odd harmonics are gradually reduced relative to the even ones, so that the perceived height of the tone moves smoothly up an octave.

In an experiment by Roy Patterson and his colleagues, a set of tones was employed, each of which consisted of the first 28 harmonics, and in which the relative amplitudes of the odd and even harmonics were varied. The subjects' task was to judge the octave in which each tone occurred. Averaging the results across subjects, when the odd harmonics were 27 dB lower than the even ones, listeners judged the tones to be an octave higher; at smaller amplitude discrepancies, averaged judgments of height fell between the higher and lower octaves.[13]

Given these findings, I surmised that one might be able

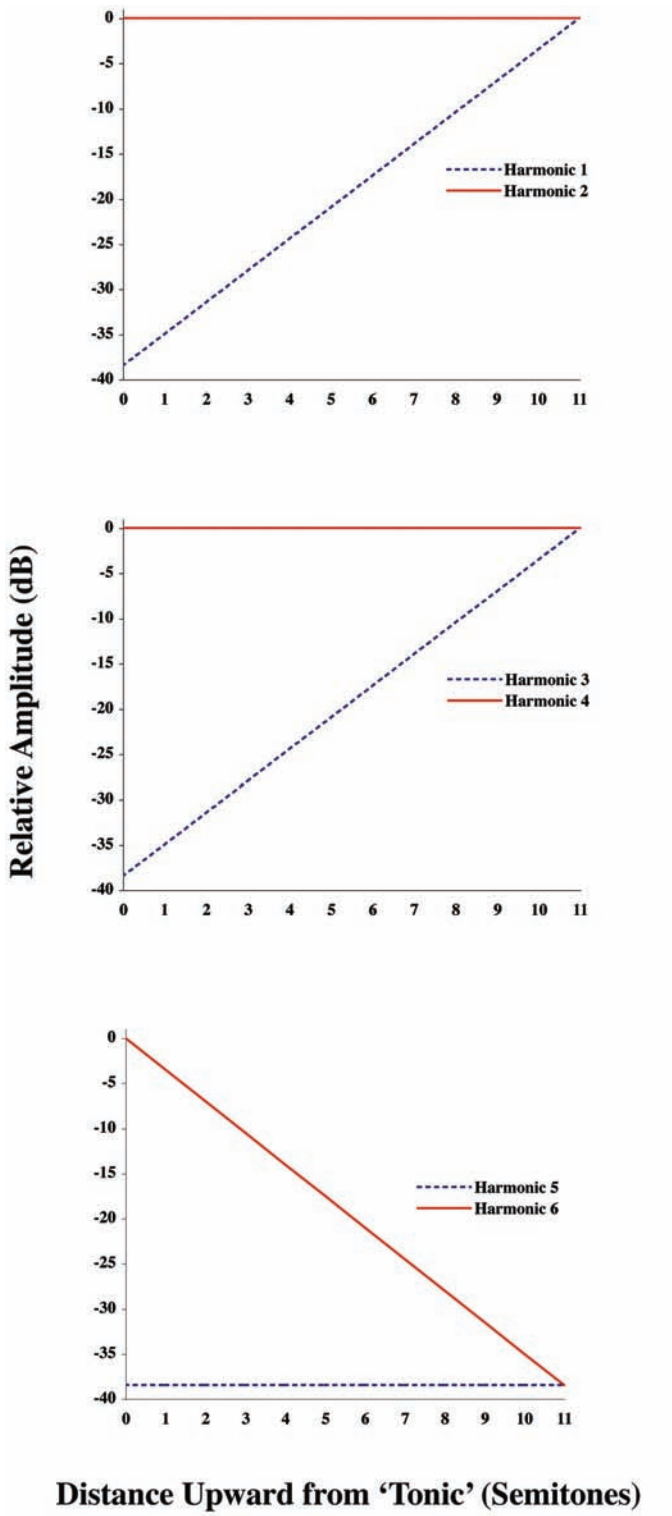

Fig. 4. Algorithm for producing pitch circularity employed by Deutsch.[16] The graphs show the progression of the relative amplitudes of Harmonics 1 and 2, Harmonics 3 and 4, and Harmonics 5 and 6, as F0 moves upward from the 'tonic' of the scale. See text for details. Reprinted from Deutsch, Dooley, and Henthorn.[16]

to generate circular banks of tones by systematically varying the relative amplitudes of the odd and even harmonics.[14] We can begin with a bank of twelve tones, each of which consists of the first six components of a harmonic series, with ***F***0s varying over an octave in semitone steps. For the tone with highest ***F***0 the odd and even harmonics are equal in amplitude. Then for the tone with ***F***0 a semitone lower, the odd harmonics are reduced in amplitude relative to the even ones, so raising the perceived height of the tone. Then for the tone with ***F***0 another semitone lower, the odd harmonics are further reduced in amplitude, so raising the perceived height of the tone to a greater extent. We continue moving down the octave in semitone steps, reducing the amplitudes of the odd-numbered harmonics further with each step, until for the lowest ***F***0 the odd-numbered harmonics no longer contribute to perceived height. The tone with the lowest ***F***0 is therefore heard as displaced up an octave, and so as higher in pitch than the tone with the highest ***F***0—and pitch circularity is thereby obtained.

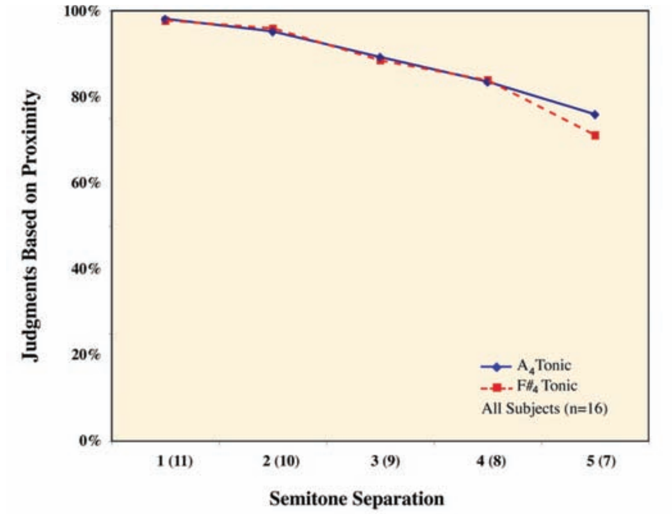

Fig. 5. Subjects were presented with pairs of tones created using the algorithm by Deutsch,[16] and they judged whether each tone pair rose or fell in pitch. The graph plots the percentages of judgments based on pitch class proximity, as a function of distance between the tones within a pair along the pitch class circle. Adapted from Deutsch, Dooley, and Henthorn.[16]

After some trial and error, I settled on the parameters shown in Fig. 4. Complex tones consisting of the first six harmonics were employed, and the amplitudes of the odd-numbered harmonics were reduced by 3.5 dB for each semitone step down the scale; therefore for the tone with lowest ***F***0 the odd harmonics were 38.5 dB lower than the even ones. To achieve this pattern for harmonic pairs 1 and 2, and harmonic pairs 3 and 4, the even numbered harmonics were at a consistently high amplitude, while the odd numbered harmonics decreased in amplitude as ***F***0 descended. To obtain the same pattern of relationship for harmonic pairs 5 and 6, harmonic 5 was consistently low in amplitude while harmonic 6 increased in amplitude as the scale descended.

In a formal experiment to determine whether such a bank of tones—hereafter referred to as a scale—would indeed be perceived as circular, my colleagues Trevor Henthorn, Kevin Dooley and I created two such scales;[15] for one scale the lowest ***F***0 was A_4 and for the other the lowest ***F***0 was $F\#_4$. (For want of a better word, we refer to the tone with the lowest ***F***0 as the *tonic* of the scale) For each scale, each tone was paired with every other tone, both as the first and the second tone of a pair, and subjects were asked to judge for each pair whether it rose or fell in pitch.

We found that judgments of these tones were overwhelmingly determined by proximity along the pitch class

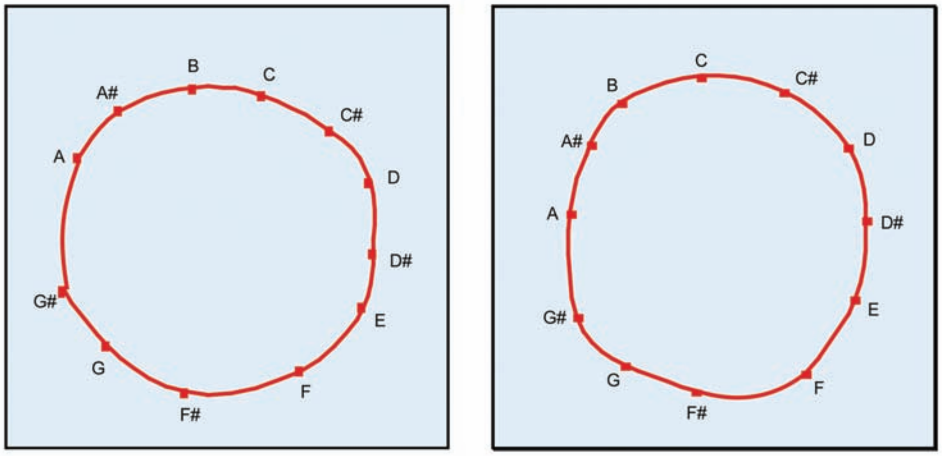

Fig. 6. Multidimensional scaling solutions produced from the relative pitch judgments of tones created using the algorithm of Deutsch,[16] made by an individual subject. The plot on the left shows the solution for tones in the scale based on the A4 tonic, and the plot on the right shows the solution for tones in the scale based on the F#4 tonic. Adapted from Deutsch, Dooley, and Henthorn.[16]

circle.[16] Figure 5 shows the percentages of judgments that were in accordance with proximity, for both scales, and averaged across all subjects. As can be seen, when the tones within a pair were separated by a semitone, proximity determined their judgments almost entirely. As with Shepard's experiment on octave-related complexes, as the tones within a pair were separated by a larger distance along the pitch class circle, the tendency to follow by proximity lessened. And even when the tones were separated by almost a half-octave, the tendency for judgments to follow the more proximal relationship was very high. When we subjected the data to Kruskal's nonmetric multidimensional scaling, we obtained strongly circular solutions, as illustrated in those from an individual subject shown in Fig. 6. We also created sound demonstrations based on this algorithm. These included endlessly ascending and descending scales moving in semitone steps, and endlessly ascending and descending glissandi, and are presented as **Sound Demonstrations 4 - 7**.

The finding that circular scales can be obtained from full harmonic series leads to the intriguing possibility that this algorithm could be used to transform banks of natural instrument tones so that they would also exhibit pitch circularity. William Brent, then a graduate student at the University of California, San Diego music department, has shown that such transformations can indeed be achieved. He used bassoon samples taken from the Musical Instrument Samples Database at the University of Iowa Electronic Music Studios, ranging in semitone steps from D#2 to D3. Using continuous overlapping Fourier analysis, he transformed the sounds into the frequency domain, and there reduced the amplitudes of the odd harmonics by 3.5 dB per semitone step downward. He then performed inverse Fourier transforms to generate the altered waveforms in the time domain. Circular banks of bassoon tones were thereby produced.[17] Endlessly ascending and descending scales employing these tones are presented as **Sound Demonstrations 8 and 9**.

It remains to be determined which types of instrument sound can be transformed so as to acquire this property. However, Brent has also achieved some success with flute, oboe, and violin samples, and has shown that the effect is not destroyed by vibrato. The Digital Signal Processing (DSP) module to produce these transformations was created for the Pd Programming environment,[15, 17] so that composers and performers can now begin to experiment with this algorithm live and in real time, as well as in recording contexts.

Hypothesized neuroanatomical substrates

What do we know about the neuroanatomical substrates underlying the circular component of pitch? An interesting study by J. D. Warren and colleagues sheds light on this issue.[18] These researchers used functional magnetic resonance imaging (fMRI) to explore patterns of brain activation in response to two types of tone sequence. In the first type, the harmonic components of the tones were at equal amplitude, but $F0$ was varied, so that pitch class and pitch height varied together. In the other type of sequence, pitch class was kept constant but the relative amplitudes of the odd and even harmonics were varied, so that only differences in pitch height were produced. Presentation of the first type of sequence resulted in activation specifically in an area anterior to the primary auditory cortex, while the second type of sequence produced activation primarily in an area posterior to this region. Based on these findings, the authors concluded that the circular component of pitch is represented in the anterior region.

Pitch circularity might, however, have its origins earlier in the auditory pathway. Gerald Langner has provided evidence in the gerbil that the ventral nucleus of the lateral lemniscus is organized in terms of a neuronal pitch helix, so that pitches are arranged in helical fashion from top to bottom with one octave for each turn of the helix.[19] This indicates that the lateral lemniscus might be the source of the circular component, and that it is further represented in the cortex.

A paradox within a paradox

There is an additional twist to the paradox of pitch circularity. We have seen that when listeners are presented with ordered pairs of tones that are ambiguous with respect to height, they invoke proximity along the pitch class circle in making judgments of relative pitch. But we can then ask what happens when a pair of such ambiguous tones is presented which are separated by a half-octave (or tritone) so that the same distance along the circle is traversed in either direction.

For example, what happens when the pattern C-F# is presented? Or the pattern A#-E? Since proximity cannot then be invoked, will such judgments be ambiguous, or will something else occur?

I conjectured that for such patterns, the auditory system would not settle for ambiguity, but would instead make reference to the absolute positions of the tones along the pitch class circle, so that tones in one region of the circle would be heard as higher and tones in the opposite region as lower. This conjecture was confirmed in a series of experiments employing Shepard tones consisting of six octave-related components, with tones within a pair generated under the same spectral envelope.[20] The experimental design controlled for sources of artifact—for example tones were generated under envelopes that were placed in different positions along the spectrum. [21,22, 23,24] Judgments of relative pitch were found to depend in an orderly fashion on the positions of the tones along the pitch class circle.

Another and entirely unexpected finding also emerged from these studies—the orientation of the pitch class circle with respect to height varied strikingly across listeners. For example, some subjects would hear the tone pair D-G# (and C#-G, and D#-A) as ascending, whereas others would hear the same patterns as descending. Then the first set of subjects would hear the tone pair G#-D (and G-C#, and A-D#) as descending while the second set of subjects would hear these patterns as ascending. Such individual differences can be easily demonstrated by presenting the four tritone pairs in **Sound Demonstration 10** to a group of listeners, and asking them to respond with a show of hands whether each tone pair ascended or descended in pitch. This demonstration is particularly striking when played to a group of professional musicians, who are quite certain of their own judgments and yet recognize that others are obtaining entirely different percepts.

In other experiments, R. Richard Moore, Mark Dolson and I studied two-part melodic patterns composed of the same octave-related complexes, and found that judgments here also depended on the positions of the tones along the pitch class circle.[25,26] In general, the tritone paradox and related paradoxes formed of two-part patterns show that while pitch height and pitch class are in principle separate dimensions, one dimension can influence the other.

Summary and conclusions

The phenomenon of pitch circularity has implications for our understanding of pitch perception, as well as for musical composition and performance. It is likely to intrigue acousticians, mathematicians, and musicians for many years to come. The experiments and sound demonstrations described here indicate that the classical definition of pitch as "that attribute of auditory sensation in terms of which sounds may be ordered on a scale extending from high to low" [27] should be amended to include the circular dimension also. The experimental decoupling of the linear and circular components of pitch provides a useful tool for the further investigation of the neural underpinnings of these two components, which are presumably processed separately at some stage in the auditory system. For musicians the development of new software that largely decouples pitch class and pitch height, and does so in real time, has opened up intriguing new avenues for composition and performance.AT

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