Computer Simulation for Predicting Acoustic Scattering from Objects at the Bottom of the Ocean

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Applying the modern discipline of computational mechanics to solving complex mathematical problems in acoustics.

Introduction

Since the early 20th century, SONAR (SOund Navigation And Ranging) has been used in military, commercial and scientific applications to help find objects submerged in the oceans, either by listening passively with underwater hydrophones to the sounds that noisy objects emit (passive sonar) or by actively projecting sound into the water and listening to the echoes reflected from quiet objects (active sonar, **Figure 1**). Sonar has been used successfully for detection (is there something out there?), localization (where is it?), some degree of classification (what type of object is it, e.g., man-made or marine organism?) and some degree of identification (what is the object?). In order to more precisely identify the nature of a detected object, an active sonar technique has recently emerged that produces additional information about the object, including not only its size and shape but also its internal composition. The resulting body of information is referred to as the "acoustic scattering signature" (Burnett, 2015), or just acoustic signature, of the object.

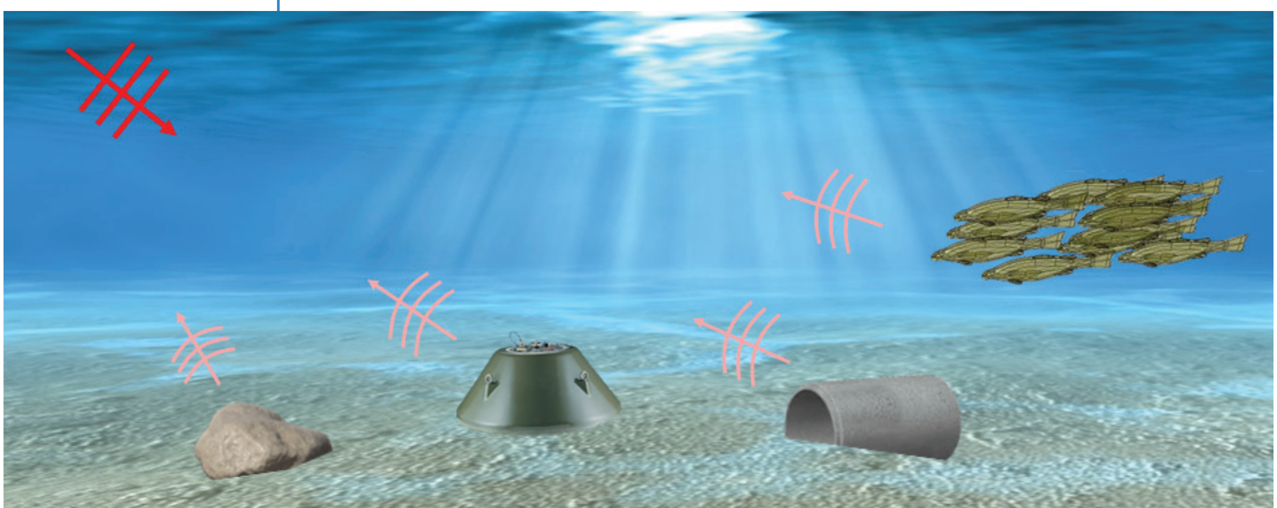

Figure 1 *An underwater scenario depicting a sonar wave (red arrow) insonifying various objects on or near the sedimentary ocean bottom – rock, mine, concrete pipe, fish school – and the resulting sound waves scattered back from the objects (pink arrows).*

The Physics Underlying an Acoustic Scattering Signature

Consider an arbitrary object in free space that is insonified by a monochromatic (single frequency) sound wave (**Figure 2**). Real objects are made of solid materials (metals, plastics, bones, flesh, etc.) which change shape or volume "elastically" when any stress is applied (albeit by microscopic amounts when small acoustic pressures are applied). The incident sound wave is an oscillatory pressure disturbance in the water. As it strikes the object, the oscillating pressure on the surface of the object induces elastic (solid) waves to propagate throughout all parts of the structure. The vibrating object will, in turn, exert an oscillating pressure on the surrounding water, which produces pressure waves in the water that propagate away from the object, so-called scattered waves (also called echoes). The scattered waves will propagate out in all directions, with different intensity in different directions. As the incident sound wave changes direction relative to the object (the angle θ in **Figure 2**) or changes frequency, the vibrations in the object will change, which, in turn, will change the scattered waves.

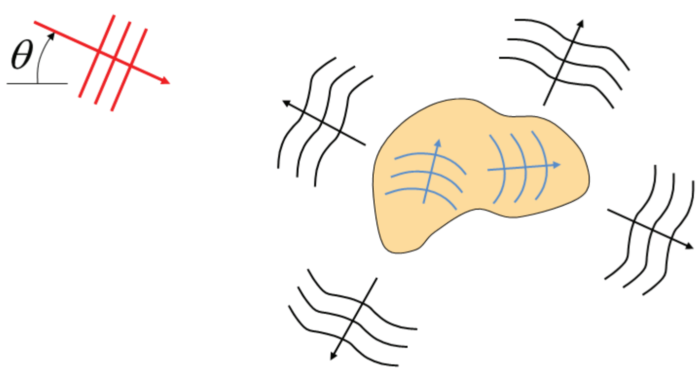

Figure 2. An object submerged in a fluid is insonified by a plane wave (red arrow), causing the object to vibrate (interior elastic waves, represented by blue arrows), which re-radiates scattered waves (black arrows) back into the fluid in all directions.

When an object (also called "target") is insonified by a plane wave (wave fronts are planar, rather than curved, as occurs when the sound source is far from the object), the intensity of the scattered pressure wave back in the same direction that the sound wave came from is expressed by the monostatic (source and observer in same direction) far-field target strength, TS:

$$TS(f,\theta) = \lim_{r\to\infty} 20\,\mathrm{Log}_{10}\left(\frac{r|p(\boldsymbol{r})|}{r_0 p_0}\right) \quad (1)$$

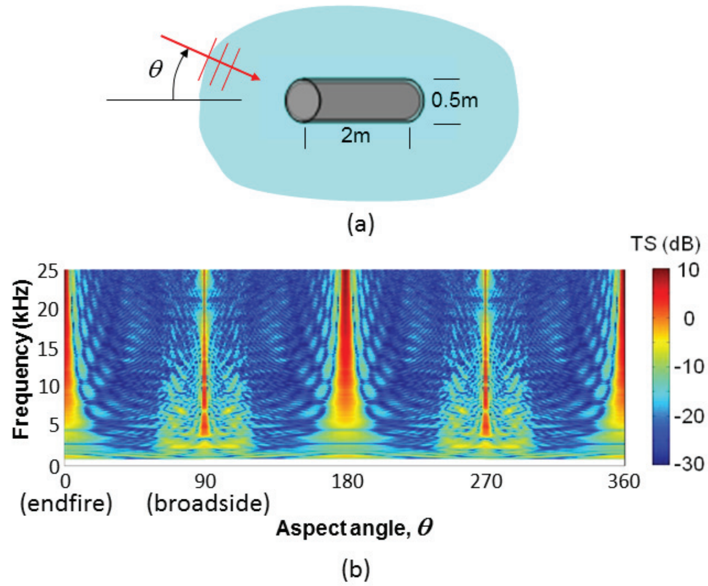

Figure 3. (a) Insonifying a steel cylindrical closed shell. (b) The resulting acoustic scattering signature template.

where f is frequency, θ is the aspect angle of the incident sound wave (defined for each different problem, but is usually the azimuthal angle, which is a horizontal angle about the vertical to the ocean bottom), $p(\boldsymbol{r})$ is the pressure of the scattered wave, | | indicates magnitude (since p is a complex-valued function), $\boldsymbol{r}$ is a position vector from the object to the "observer" (where the TS is being measured), r is its magnitude, and r_o and p_o are a reference distance and pressure that normalize and make dimensionless the argument of the logarithm. In 3-dimensional space, $p(\boldsymbol{r}) \propto 1/r$ far away from the object, in the so-called far field; hence the numerator, $r|p(\boldsymbol{r})|$, converges to a limiting value as $r \to \infty$. The symbol $\lim_{r\to\infty}$ means evaluate the scattered pressure far enough away to obtain the limiting value. The units of TS are decibels (dB).

An acoustic scattering signature is the target strength of an object, **Equation (1)**, that has been insonified over a broad band of frequencies and, for each frequency, over a broad range of aspect angles. The resulting values of TS are plotted as a template (terminology from radar) of TS vs. f and θ, with TS displayed as a color. This is illustrated below for a steel cylindrical closed shell, with a color bar on the right to quantify the TS values (**Figure 3**).

One can see that there is a great deal of information about the object in such a template. For example, the geometry of object and incident sound wave in part (a) of **Figure 3** is clearly symmetric about the incident directions that are parallel to the axis of the cylinder (endfire, $\theta = 0°$) and perpendicular to the axis (broadside, $\theta = 90°$), and this is manifested in corresponding symmetries in the template in part (b).

The TS is also strongest (red color) in these two directions because there are parts of the surfaces of the shell that are oriented perpendicular to the incident wave and therefore reflect part of the energy directly backward; at other angles the surfaces are oblique, reflecting energy in other directions. Other intense areas of the template (yellow, green, light blue) are caused by resonances of various types of internal elastic waves.

Acoustic Signature vs. Imaging

The dimension*less* frequency band of acoustic signatures is the same for all objects, no matter how large or small: from about $ka = 1$ to about $ka = 30$, where ka is dimension*less* frequency, k is wavenumber ($2\pi/\lambda$), λ is the wavelength of the insonifying plane wave (recall **Figure 2**) and **a** is an approximate "radius" of the object, namely, an average "radius" of chunky objects or shorter "radius" of long objects. At the lower frequencies, wavelengths are comparable to the exterior dimensions of the object; at the higher frequencies wavelengths are comparable to smaller interior dimensions. Equivalently, this is the range of the lower natural modes of vibration of the object. To illustrate, the vertical frequency axis in **Figure 3**, using the speed of sound in water as 1500 m/s and $a = 0.25$m, ranges from $ka = 1$ to $ka = 26$.

Sonar classification has traditionally relied on much higher frequencies, where wavelengths are very small relative to the object and therefore can produce a rough image of the object, that is, its external shape, but it cannot reveal internal composition because the shorter wavelengths can't penetrate very deeply before they die out due to attenuation per wavelength. Consequently, acoustic signatures and imaging complement each other in modern sonar systems. A key difference between the two approaches is the amount of information needed to interpret sonar output: imaging produces a picture of the object so an engineer can identify the object simply by looking at the image, whereas a signature produces only a TS template, which requires a computer to detect meaningful patterns in the template.

The Need for Computer Simulation

One approach to the computer search for such patterns is to see if there are similarities with the templates of similar objects in a variety of realistic configurations, for example, rest ing on the bottom, partially buried, fully buried, tipped, etc., or in different types of sediment such as sand, clay, mud, etc., as well as objects with similar construction, for example, decoys or manufacturing variations. All of these variations can have a significant effect on the vibrational response of the object and hence its acoustic signature. That requires having a large library of reference acoustic signature templates.

To construct such a library, one could perform experiments on actual objects. But experiments are expensive and time consuming so only a few can be performed, and one cannot perform experiments on unavailable objects or environments. Computers, however, can model virtually any object/environment scenario of interest, including nonexistent scenarios. The cost of computer resources per model is negligible compared to that of a real underwater experiment and often faster by orders of magnitude, sometimes enabling hundreds or thousands of templates to be computed in the same time as performing one underwater experiment. There is clearly a need for a computer simulation system that is both high-fidelity and computationally fast.

Computer Simulation

The Naval Surface Warfare Center Panama City Division (NSWC PCD) has developed a high-fidelity, broadband (CW analyses), three-dimensional (3-D), finite-element (FE) computer simulation system called PC-ACOLOR (Panama City-Acoustic COLOR), which models the acoustic scattering signature (also called acoustic color) of single or multiple realistic targets at the bottom of the ocean.

The principle challenges to developing such a system were as follows:

- Multiscale spatially: From small details in the objects (cm) to large distances in the ocean (km).
- Broadband: A five-octave range, $ka \approx 1$ to 30.
- A need for extraordinarily high computational efficiency: One acoustic signature template requires sweeping typically over several hundred frequencies, and, for each frequency, several hundred aspect angles, requiring $O(10^5)$ 3-D models. Modeling techniques developed by the author since the 1980s have enabled the code to currently compute about one template per day, an adequate pace for applications so far. Nevertheless, the next level of applications will require computing several hundred templates per day in order to create statistically robust acoustic signature libraries. R&D to achieve such speeds is underway and should be operational when this article is published (see The Way Forward at the end of this article).

The next three subsections describe the approach used by NSWC PCD: the physics, the governing mathematics, and the computational modeling technique.

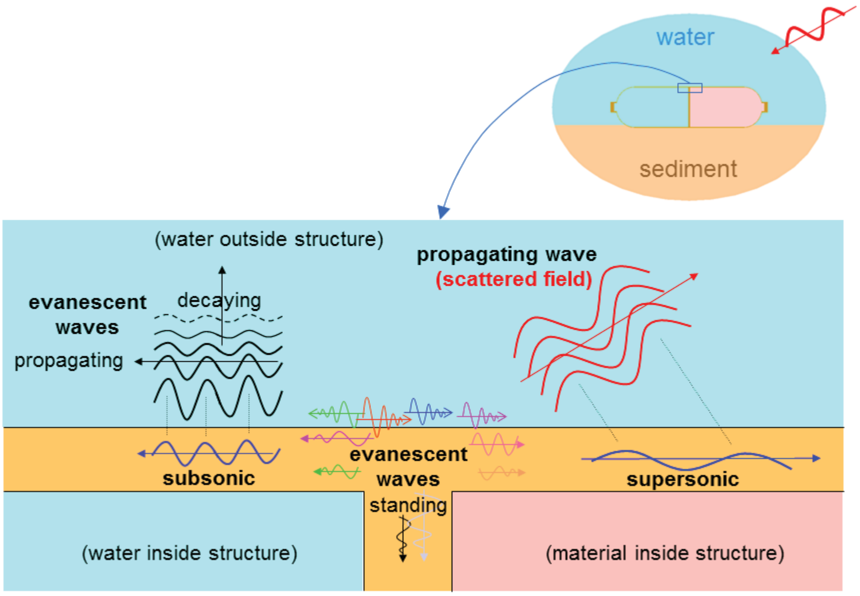

Figure 4. A concept sketch illustrating how both evanescent and propagating waves exist in the elastic structure and in the surrounding fluid.

It Begins With the Physics

High fidelity means high accuracy relative to reality, which means the inclusion of many structural details in the objects and, equally important, all the physics necessary for describing all possible types of wave motion in the objects and surrounding fluids. This includes not only the many types of propagating waves that are described in most textbooks (e.g., Lamb waves, Love waves, creeping waves, etc.) but also so-called evanescent waves, which are rarely mentioned in textbooks.

Evanescent waves exist at the interfaces between different materials and generally have shorter wavelengths than propagating waves (Mindlin, 1960; Zemanek, 1972). They are spatially decaying in one or more directions and are standing waves in the directions of decay. Their spatial decay is not due to material damping or geometric spreading. Nature needs such waves to satisfy continuity of physics at material discontinuities or near small-scale features where the wavelengths of propagating waves are too long to resolve those features. (The word "evanescent" is misleading, implying temporal decay (ephemeral, fleeting); however, the decay is spatial, not temporal.)

Evanescent waves are especially important at the fluid/solid interface ("wet surface") of targets since this is where scattered waves are launched into the fluid. **Figure 4** illustrates these concepts with a sketch of an underwater manmade structure, consisting of a thin-shell container with an internal partition. It zooms in on a typical small structural discontinuity – in this case, the intersection of the outer shell and the internal partition – showing both propagating and evanescent waves and especially those "trapped" on the outer wet surface of the target, which are evanescent normal to the surface while propagating parallel to the surface.

Figure 4 illustrates a very important aspect of the small-scale/large-scale interaction in structural acoustics that is often over-looked in the literature on computer modeling of such phenomena: small-scale local evanescent elastic waves inside the structure can have a significant effect on the propagating acoustic field (scattered field) far away, the latter usually being the goal of computer modeling of target scattering. Thus, the amplitudes and phases of the local elastic waves near a shell/partition intersection affect the amplitudes and phases of the propagating elastic waves along the length of the shell, and these, in turn, affect the amplitudes and phases of the propagating acoustic waves (the scattered waves) launched from the wet surface into the fluid. All these interactions will vary as a function of frequency and aspect angle of the incident wave.

In short, high fidelity modeling requires an approach that captures the full 3-D nature of the complicated wave fields near structural discontinuities as well as the wide range of wavelengths associated with evanescent and propagating waves, even for single-frequency (CW) models. To this end, PC-ACOLOR employs 3-D physics throughout all solids and fluids; no engineering approximations are made anywhere.

The Governing Mathematical Equations

The PC-ACOLOR system is frequency-domain (also called monochromatic, steady state or CW, the latter meaning continuous wave) rather than time-domain, for several reasons, foremost being (i) analyses are 3-D rather than 4-D, (ii) parallel computation using distributed processing is more easily facilitated, (iii) material properties (e.g., frequency-dependence and attenuation) are more readily available, and, of course, (iv) templates are in the frequency domain. When time-domain solutions are desired, such as for comparison with time-series responses from experiments, they are computed by inverse Fourier transforming the broadband frequency-domain solutions.

PC-ACOLOR separates the analysis into a local analysis in the "target region," which is analyzed using partial differential equations (PDEs), and a global analysis in the "exterior to target region," which is analyzed using an integral equation. The separation introduces no approximations to the 3-D physics.

Target Region:

The region for modeling local scattering from targets, the so-called "target region," comprises the targets and the fluids surrounding the targets out to an ellipsoidal boundary circumscribing the targets and separated from them by approximately half the characteristic wavelength of the fluids (**Figure 5a**). Inside the target region PC-ACOLOR finds a solution to two partial differential equations (PDEs) – one for fluids and the other for solids – that describe all phenomena within linear acoustics.

The PDE for describing monochromatic (single frequency) sound waves in fluids is the Helmholtz equation (also called the monochromatic wave equation),

$$-\nabla \cdot \left(\frac{1}{\omega^2 \rho} \nabla p \right) - \frac{1}{B} p = 0 \qquad (2)$$

where p is the scattered pressure field, $\omega = 2\pi f$, f is frequency, and B and ρ are the bulk modulus and density, respectively, of the fluid.

The PDE for describing monochromatic elastic waves in solids is the elastodynamic equation,

$$-\nabla \cdot (\boldsymbol{c} \nabla \boldsymbol{u}) - \omega^2 \rho_S \boldsymbol{u} = \boldsymbol{f}_S \qquad (3)$$

where $\boldsymbol{u}$ is the (vector) displacement of a particle of the material as the wave passes by, $\boldsymbol{c}$ is a 4th-rank tensor of elastic moduli, ρ_S is the density of the solid material and $\boldsymbol{f}_S$ is an applied force (vector) per unit volume.

In addition, conditions for continuity of normal stress and normal displacement are applied on fluid/solid interfaces and radiation boundary conditions (rad BCs), which contain the physics for the large exterior region, are applied to the ellipsoidal boundary (Burnett, 2012).

Equations (2) and **(3)**, combined with the continuity conditions, radiation boundary conditions and applied excitations (e.g., incident acoustic field), constitute a well-posed mathematical problem for which there exists a solution valid anywhere inside the target region.

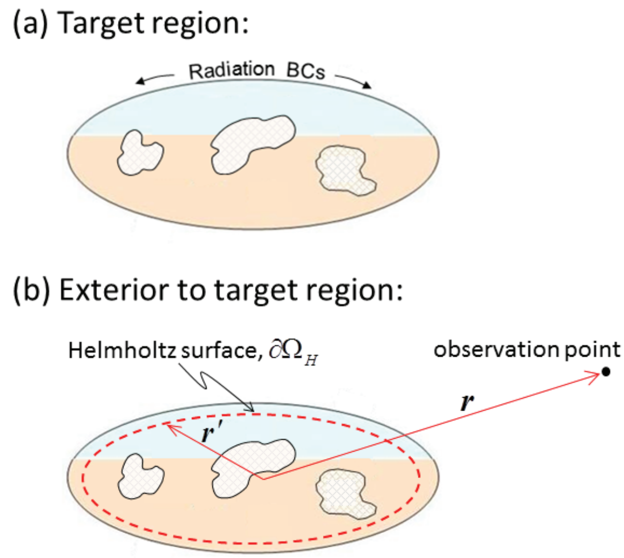

Figure 5. *Different governing equations are used for computing the scattered field inside the target region (Figure 5a) and exterior to the target region (Figure 5b).*

Exterior to Target Region:

The solution anywhere outside the target region is computed using the Helmholtz integral,

$$p(\boldsymbol{r}) = \iint_{\partial\Omega_H} \left(\frac{\partial G(\boldsymbol{r},\boldsymbol{r}')}{\partial n} p(\boldsymbol{r}') - G(\boldsymbol{r},\boldsymbol{r}') \frac{\partial p(\boldsymbol{r}')}{\partial n} \right) d\Gamma \qquad (4)$$

where the integration is over a mathematical surface, $\partial\Omega_H$, called the Helmholtz surface, that circumscribes the objects (**Figure 5b**). The quantities $p(\boldsymbol{r}')$ and $\partial p(\boldsymbol{r}')/\partial n$ are the scattered pressure and its derivative normal to the surface, respectively, which are the solution to **Equation (2)** inside the target region. $G(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function for the environment, which describes how sound waves propagate in the large ocean environment in the absence of the target. It can sometimes be expressed with simple formulas for simple idealized environments but can also be computed numerically in a separate computer simulation for more complicated realistic environments. The pressure $p(\boldsymbol{r})$ computed from **Equation (4)** is the pressure $p(\boldsymbol{r})$ in **Equation (1)**.

The Computational Modeling Technique

It is impossible to obtain an exact solution to **Equations (2) – (4)** for virtually any realistic objects using classical methods of applied mathematics unless one simplifies the equations a great deal, thereby eliminating much of the physics. Fortunately, that is not necessary, as the branch of modern mathematics known as finite-element (FE) analysis can produce an approximate solution that is as close as desired to the exact solution without simplifying any of the physics.

FE analysis is an extension of classical calculus (Burnett, 1987). It began in the mid-20th century and has grown rapidly, becoming such a powerful theoretical/numerical technique that it can find solutions to virtually any differential or integral equations that model (simulate) applications of almost any complexity. It has often been described as the most significant revolution in applied mathematics in the twentieth century, a perception that this author, who has worked with FE analysis for almost half a century, can heartily agree with. The merging of FE analysis with computer technology – two sciences that evolved concurrently and synergistically – has created the modern discipline known as computational mechanics (IACM, 2014). As computers continue to evolve, the power of FE analysis continues to grow apace. This article on predicting acoustic signatures is just one illustration of the power of modern computer simulation.

The essence of FE analysis is to subdivide the domain of a mathematical problem into a mesh of very small, simply shaped "elements" and then to approximately represent **Equation (2)** or **(3)** inside each and every element by transforming the differential equations into approximately equivalent algebraic equations. The algebraic equations in adjacent elements are interrelated, producing a continuity of physics across all the elements. Consequently, all of the element equations are coupled together into a very large system of simultaneous algebraic equations, typically hundreds of thousands or millions or even billions, which must then be solved on a computer. As elements in a mesh are made progressively smaller, or the mathematical representation inside each element is enriched, the FE approximate solution becomes progressively more accurate, converging eventually to the exact solution of the original mathematical problem.

The FE modeling process is illustrated below for the problem of acoustic scattering from a spherical steel shell resting on (almost touching) a fluid-like sandy ocean bottom. In this model, the very large ocean and sediment regions can be considered mathematically as "infinitely large" regions (**Figure 6 (a)**). To reduce the computational size of this problem, the large water and sediment regions can be replaced by a small surrounding ellipsoid (or spheroid or sphere) of water and sediment, with all the physics in the removed regions represented instead by mathematical relations, known as radiation boundary conditions, or rad BCs, applied to the outer boundary of the ellipsoid (**Figure 6 (b)**). This reduced model can be reduced further, to just one quadrant, by dividing it by any two perpendicular vertical planes intersecting the center of the spherical shell (**Figure 6 (c)**). One can then analyze just the one quadrant by decomposing all acoustic fields into components that are symmetric or antisymmetric with respect to those planes, in a way that preserves all the physics in the reduced model. The last step in the modeling process is to create a computationally efficient FE mesh for the quadrant model: larger elements away from the shell to represent long-wavelength propagating waves, smaller elements near to the shell to represent shorter-wavelength evanescent waves, and even smaller elements in the gap between the shell and the ocean bottom (**Figure 6(d)**).

In addition to the above computational efficiencies, the model is scaled with respect to frequency: as frequency increases, the ellipsoidal outer boundary moves closer to the targets in order to maintain a separation of about half a wavelength at all frequencies. This yields the additional advantage of maintaining approximately uniform modeling error across the frequency band.

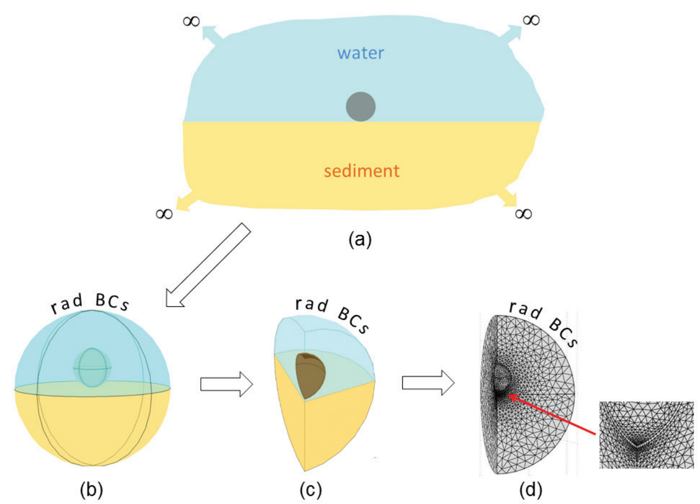

Figure 6. *(a) Full model: A spherical steel shell resting on the ocean bottom. The infinity symbols (∞) imply the water and sediment each occupy an infinite "halfspace". (b) Reduced model. (c) One quadrant of the reduced model. (d) The FE mesh for the quadrant model.*

In summary, the computer modeling process consists of using FE analysis to find numerical solutions to **Equations (2)** and **(3)** that describe the physics of wave propagation in fluids and solids, and then using **Equation (4)**, in conjunction with those solutions, to find the scattered acoustic pressure anywhere in the ocean. This process is repeated over and over for different frequencies and different aspect angles. Inserting those scattered pressures into **Equation (1)** yields the sought-after TS as a function of frequency and aspect angle, which is the acoustic signature of the object.

Verification & Validation

In the modern world of computational science, computer simulation has evolved to become the third essential research methodology, alongside theory and experiment (Oberkampf, 2002; Post, 2005). Since computer simulation is prone to human errors throughout the development and modeling process, it is important, in order to achieve reliable solutions, to continually subject models to a process of experimental validation (of the physics) and numerical verification (of the mathematics), also known as V&V (**Figure 7**). Ideally, this is a continuous, never-ending process.

Following are two examples, one of verification and one of validation. Both examples illustrate an essential feature of V&V: a comparison of two different approaches to a problem. Each approach has strengths and weaknesses and is prone to human error, so neither solution is certain. Thus, V&V is a two-way street, each approach providing increased confidence in the other. As more and more V&V testing is done, it might be said that one's confidence level can approach certainty asymptotically!

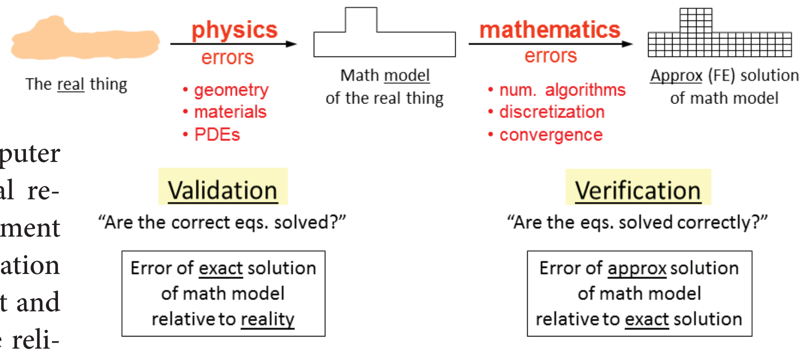

Figure 7. Validation of physics and verification of mathematics.

Verification

Verification is often accomplished by comparing two models using *independent* modeling techniques and looking for agreement over a broad range of the variables (not just at a few isolated spots). The rationale is this: two modeling techniques that use different methods are unlikely to produce the same errors over a broad, continuous span of data; ergo, if both solutions agree, then there are probably no errors, so both have probably found the correct solution.

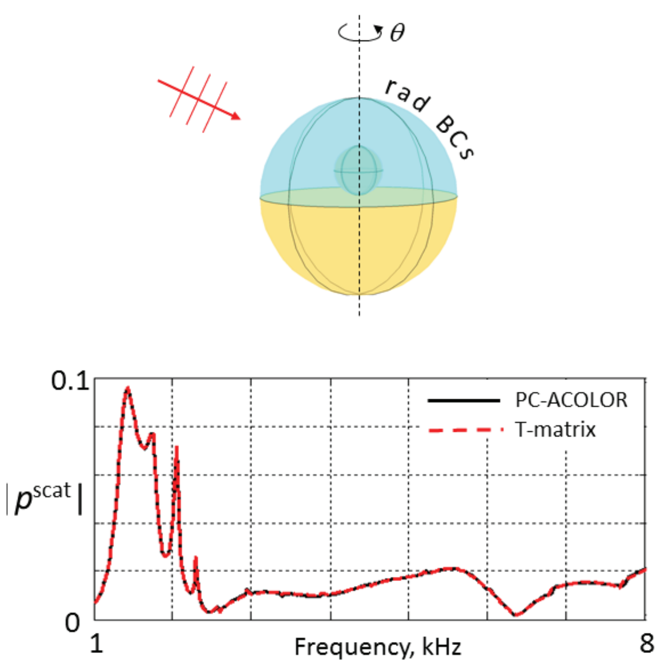

***Figure 8.** Verification of computer simulation of scattering from spherical shell resting on sediment on ocean bottom, vis-à-vis an independent analytical technique (T-matrix method).*

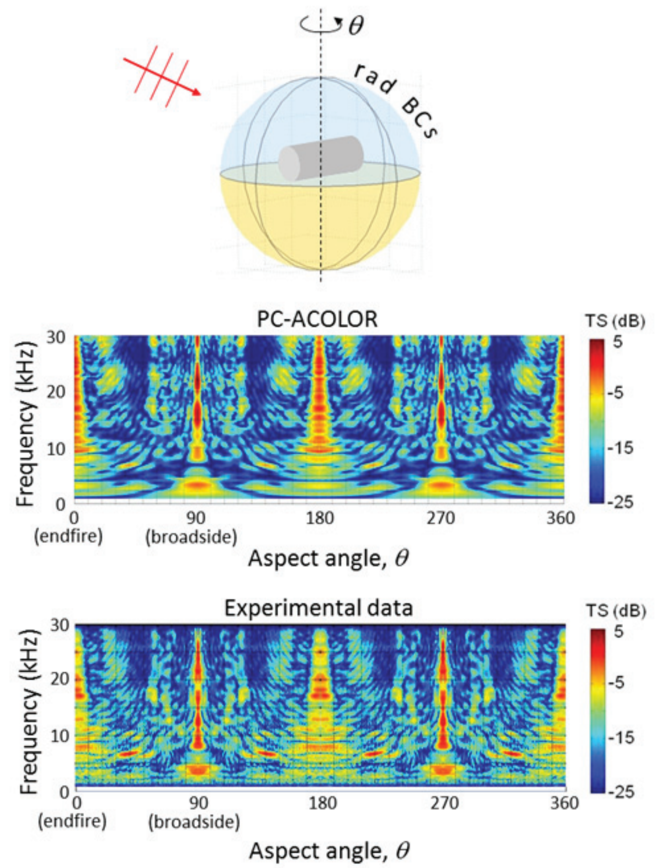

***Figure 9.** Validation of computer simulation of scattering from a solid aluminum cylinder resting on sediment, vis-à-vis data from an in-water experiment. (Experimental data exhibit noise at the low frequencies due to a low signal-to-noise ratio for the equipment.)*

Figure 8 illustrates verification vis-à-vis a non-FE analytical method, for the problem of scattering of a plane wave from a spherical steel shell resting on the sediment (cf. **Figure 6**). The geometry of a sphere is very simple, so this problem is amenable to other, non-FE solution techniques, such as the T-matrix method, which is limited to very simple geometric shapes. The mathematical formalism and computer codes are completely different for FE and T-matrix analyses.

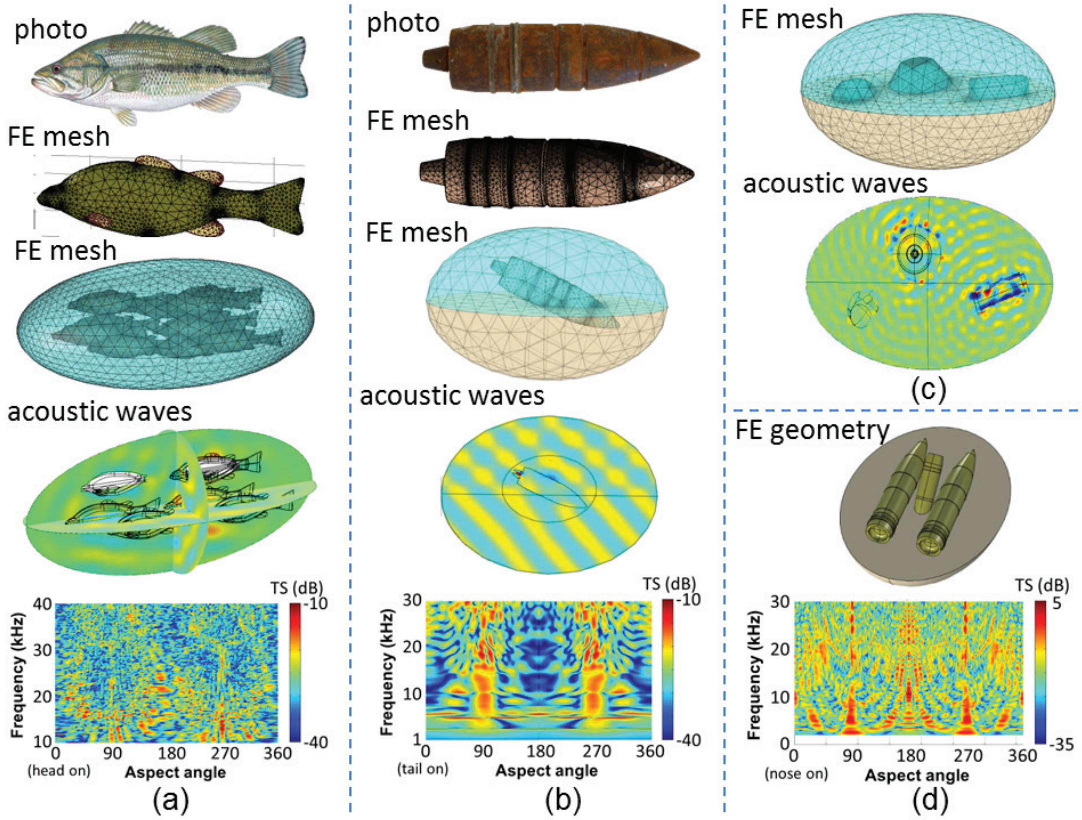

Figure 10. (a) Fish and school of fish (cf. Figure 1). (b) Unexploded artillery shell partially buried in sediment. (c) Rock, mine and concrete conduit pipe on sediment (cf. Figure 2). (d) Unexploded Howitzer shells on sediment and bullet partially buried.

Since the sphere and its environment are axisymmetric about the vertical (dashed line), the backscattered pressure is independent of the aspect angle, θ, so verification only needs to be done as a function of frequency. The two solutions in **Figure 8**, for the magnitude of the backscattered pressure vs. frequency, agree to about 3 significant figures over the entire three octaves of frequency, including the sharp spikes. Such strong agreements provide confidence in both mathematical techniques.

Validation

Validation is accomplished by comparing computer model predictions with data measured in experiments with real objects. This tests whether the correct physics is being used in the computer models (cf. **Figure 7**). For example, are **Equations (2)** and **(3)** adequate to capture all phenomena of interest or are additional equations necessary? Are there physical features in the real object that were intentionally omitted from the model to simplify the modeling, but perhaps shouldn't have been omitted? Has all the experimental equipment been calibrated properly? Validation tests the physics in the model against the real world, which involves ranges of uncertainty in physical properties, experimental imprecision, etc. Therefore, accuracies are generally much lower than those for verification; agreements to within a few dB are often considered very good.

Figure 9 illustrates validation vis-à-vis experimental data, for the problem of scattering of a plane wave from a solid aluminum cylinder resting on the sediment (Williams et al, 2010). There is very good agreement (to within about 3 dB) over almost the entire range of frequencies and aspect angles. This provides some confidence that the computer simulation is based on the correct physics.

Some Models of Realistic Objects

This article has used simple objects – spheres and cylinders – to explain and illustrate the physics, mathematics and FE concepts for the computer simulation of acoustic scattering from submerged objects. This concluding section shows a variety of images from models of more realistic objects, all of which used the above-described techniques. In addition to these, much more complicated underwater structures have also been modeled, which have included a considerable amount of interior structural detail.

The Way Forward

The current PC-ACOLOR computer simulation system takes one or two days to compute a typical high-fidelity acoustic signature template using a dedicated in-house rack computer with 25 quad-core processors. This pace has been adequate for several years, but the uniqueness of this type of simulation has spawned an interest in using this technology in a more aggressive way: doing massive parametric studies of large varieties of possible real-world target/environment scenarios, in order to create statistically robust acoustic signature libraries. Having such a large data base of signatures will help improve the reliability of sonar systems trying to identify an object, irrespective of small variations in construction or different types of surrounding environments. This will require tens of thousands or even hundreds of thousands of templates. To produce that many in a reasonable time will require computing at least several hundred templates per day, or a computational speed of only a few minutes per template.

R&D at NSWC PCD has just recently achieved such a speed, while preserving all the same 3-D high-fidelity physics in the simulation mathematics! In addition, the radically new system will be ported to a High Performance Computing (HPC) center with several thousand processors so that it is realistic to expect that these high-fidelity simulations will soon be performed at a speed of only a few seconds per template, or thousands of templates per day. The future of this work, and its value to the U. S. Navy mission, looks very exciting indeed.

Acknowledgments

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Biosketch

Dave Burnett is the U.S. Navy's Senior Technologist for Computational Structural Acoustics. He holds 25 U.S. and international patents in the fields of computational acoustics and electromagnetism and is the author of two books on finite element analysis. Springer/ASA Press recently approved his writing a new book on computational structural acoustics. He is a Fellow of Bell Labs (formerly the R&D division of AT&T) and of the Acoustical Society, an associate editor for JASA, and an editor for the Journal of Computational Acoustics. He holds BS, MS, and PhD degrees from Cornell, Caltech and U. C. Berkeley, respectively.

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