**THE PARAMETRIC ARRAY AND LONG-RANGE OCEAN RESEARCH**

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**Introduction**

The parametric array (PA) is a nonlinear transduction process that can generate a narrow beam of low frequency sound in a medium, through the interaction of co-linear, intense, high frequency sound waves, called pump waves. The unique characteristic of a parametric array is found in its extremely narrow directivity pattern (1°-3° angular resolution) for low frequency acoustical signals. The effective width of the directivity pattern remains practically constant over a wide range of signal frequencies. The parametric array has become essentially a virtual acoustic end-fire array that has been formed in the medium (water) by the non-linear interaction of the two high frequency waves at their sum and difference frequencies (Fig. 1). As a result, it radiates a sharp, low-frequency, directional signal at the interaction frequency of its pump waves that propagates independently of the pump waves. Due to the non-resonance property of the low-frequency signal generation the parametric array can provide a sounding signal transmission in extremely wide frequency bands (more than two octaves).

The nonlinear interactions of sound waves are described by Burgers equation:

\[
\frac{\partial u}{\partial x} - \alpha \frac{\partial u}{\partial y} = \delta \frac{\partial^2 u}{\partial y^2} 
\]

where \(u\) is the fluid velocity perturbations due to sound waves, \(x\) is the space coordinate, \(y = t - x/c_0\) is the retarded time, \(\alpha = \varepsilon/c_0\), \(\varepsilon\) is the nonlinear parameter of the fluid, \(c_0\) is the sound velocity, and \(\delta\) is the dissipation coefficient that is independent of frequency. The second term in the left-hand side and the term in the right-hand side take into account the nonlinear effects such as combination frequency generation and attenuation of the wave, respectively.

The ratio of the nonlinear term of the Burgers equation

![Diagram of parametric array signal generation](image)

**Fig. 1. Parametric signal generation:** 1—acoustic array that generates two co-linear high-frequency, intense sound waves in the near field volume; 2—far field volume where parametric, low-frequency acoustic signals are generated. Directivity of parametrically generated, low-frequency signals are defined by the squared directivity pattern of high frequency pump, \(D_\theta(\Theta) - D_\theta(\Theta)\), where \(D_\theta\) is the directivity of the parametric signal and \(D_\theta\) is the directivity of the two primary signals.
to the dissipative term is the Reynolds number, $Re$. Thus:

$$
\frac{\alpha u}{\delta} \frac{\partial u}{\partial y} - \frac{\partial}{\partial y} \left( \frac{\alpha u_0}{\delta} \right) = Re \omega,
$$

(2)

where $u_0$ is the sound wave amplitude, and $\omega$ is the circular frequency of the wave. If $Re > 1$ then the nonlinear effects in a sound wave dominate the dissipation, and nonlinear effects are essential in this case, while $Re < 1$ means that nonlinear effects are weak and the wave decays before nonlinear distortion develops. One can see that the Reynolds number $Re$ increases with sound amplitude and decreases with wave frequency.

Historically nonlinear acoustics experiments were conducted in ultrasonic frequency ranges with large wave numbers and therefore it was necessary to apply very high sound intensities to observe nonlinear effects.\(^3\) Thus, nonlinear acoustics became a synonym of high intensity acoustics, with many practical applications that can be found in science such as medical ultrasound.\(^4\) But one could increase the Reynolds number without the sound intensity increasing by decreasing the frequency of the sound. Therefore, nonlinear effects could be realized for low frequency acoustics in the atmosphere or the ocean.

Because long-range ocean investigations require low frequency sound signals that can propagate long distances without severe attenuation,\(^5\) the usefulness of the parametric array for this purpose bears discussion. The main question that arises when comparing specific features of parametric and conventional acoustic arrays is: could the low-energy efficiency of parametric arrays be compensated for by its sharp directivity pattern and wide frequency band?

Long range ocean sounding by a parametric array

Parametric arrays are widely used in marine research for examining the layers under the bottom surface however, the PAs acoustic characteristics may also make it “a perfect tool for ocean acoustics.”\(^3\) Long range acoustic propagation in the ocean is characterized by strong mode coupling.\(^4\) Resolving the travel time variability in several tenths of milliseconds for multi-paths in the ocean waveguide usually requires sophisticated signal processing techniques or single mode excitation. Parametric array experiments have shown that its directivity pattern can be very sharp (1°-3° in angular resolution) and almost independent of the wave frequency. It is possible therefore, that parametric arrays may provide the broad frequency band, single mode acoustic source needed for propagation in shallow water waveguides.

To our knowledge, there was only one actual long range ocean experiment using a parametric array for up to 1000 km range signal propagation.\(^7,8\) This experiment was performed in the early 1990s during the cruise of the Russian research ships R/V Academician B. Konstantinov (ABK) and R/V Academician N. Andreyev (ANA) in the region of Kamchatka and Kuril in the Pacific (Fig. 2). The 6 m long and 2 m high side-looking array in the bow of the ABK was used as a parametric array with pump wave power of 20 kW at a mean frequency of 3 kHz. This array transmitted parametric signals in a frequency range of 230-700 Hz. The vertical pump wave directivity pattern had a width of 12° in the main lobe and its
axis was inclined from the horizon at the same angle, 12° allowing some energy of pump transmission to be trapped by the ocean waveguide. The horizontal directivity width for the pump transmission was approx 4° in the main lobe. The directivity pattern of parametric signal as well as propagation characteristics of sharply directed parametric radiation through synoptic ocean vortices were measured. Harmonic sound signals of 60 sec and 180 sec duration were generated parametrically at frequencies of 230, 400 and 700 Hz. Linear frequency-modulated signals with chirp in a frequency band of 230-700 were used as well. The angular directivity pattern was measured to be almost constant in the frequency range of signals used and close to the squared pump wave angular pattern (Fig. 3). The average level of parametric signals as a function of frequency measured (Fig. 4) showed that the efficiency of parametric array increased with signal frequency. This feature of the parametric array led to an interesting result—leveling of the spectral components of the intensity with distance (Fig. 5). In this experiment, it is seen that the signal levels for a frequency range from 230 Hz to 600 Hz are similar at a distance of about 600 km from the source.

The ABK sonar was used to investigate synoptic eddy structure at far distances. A region of the Kuril Strait with typical ocean eddies was chosen for this experiment. The ABK (transmitting) and the ANA (receiving) were spaced at a distance of 400 km apart in this area. The chain of ocean eddies was located in the ocean between the vessels. A parametric signal frequency of 700 Hz (with pump frequencies of 3.6 kHz and 2.9 kHz) was transmitted from the ABK when it passed five areas corresponding to positions of the ANA indicated in Fig. 2. The map of ocean surface temperature in this region is also shown in this figure from a satellite view. The angular pattern for the parametric signal that passed through an inhomogeneous ocean has been investigated in this research (Fig. 6). The diagrams in Fig. 6 differ drastically from the directivity pattern measured in a quiet part of the ocean when there were no eddies (Fig. 3). Additional lobes appeared and the angular distribution of these lobes changed with the position of the receiving vessel in the area of experiment. The directivity pattern remained sharp while the signal passed mainly in a quiet area (Fig. 6-N1) and completely dispersed when it propagated through an inhomogeneous current that was produced by eddies. It should be noted that the noise level in these experiments never exceeded -30 to -25 dB.

**Single-mode frequency dispersion for a parametric signal in a marine waveguide**

In shallow water, the sound field usually consists of a series of modes exhibiting frequency dispersion of the speed of propagation of a signal. The value of the dispersion depends, among other causes, on the vertical sound speed profile. The frequency dispersion provides either a spread in time of short broadband pulses that travel long distances, or concentration of acoustic signal energy within a short time interval when the frequency modulation of the signal corresponds to the dispersion conditions in the medium. In the latter case, focusing of the acoustic signal or the signal compression in time should be considered. A parametric array signal has been used to observe the compression effect.9 The acoustic signal in this experiment was radiated by a narrow-beam parametric array. The sea depth at the site of the exper-
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The parametric array was constructed in the form of a mosaic of radiating elements, half of which transmitted a high-frequency pumping signal at one frequency, and the other half transmitted a signal at a slightly different frequency. The average radiation frequency (pumping frequency) was 150 kHz. The difference frequency, the signal of interest, was within 5–20 kHz. The power of the array amplifier was 1 kW for each of the pumping frequencies. The receiving array was constructed in the form of a vertical chain of eight hydrophones, spaced at 0.25 m and mounted on a metal rod. The rod was positioned vertically at the bottom so that the chain of hydrophones covered the whole waveguide. A sequence of pulses was transmitted. The duration of a single pulse was 2 ms, and the interval between pulses was about 300 ms. Signals were simultaneously received from all of the individual hydrophones of the vertical array. The measurements were carried out for transmitter–receiver distances of 1 to 5.6 km.

The frequency–time characteristic for pulses with duration of 2 ms and a carrier frequency linearly modulated within 7–15 kHz that was propagated through the shallow water waveguide was investigated. The received signals from the vertical chain of hydrophones at a distance more than 1000 m from the source show that the major part of the energy was concentrated in the middle of the waveguide. A detailed analysis shows that the signals received by different hydrophones of the receiving array were in-phase throughout the whole waveguide depth, which indicates that there was a predominance of the single-mode propagation of the signal. Thus, under the given experimental conditions, the parametric array excited the lowest mode of the waveguide.

The signal frequency sweep was in the direction from low to high frequencies, which corresponded to normal waveguide dispersion, i.e., to the case where the group velo-

Fig. 6. Directivity patterns of the parametric array as the two ships took measurements in the inhomogeneous section of the ocean. Refer to Fig. 2 for the actual physical location of the ships. Transmitting ship (1) R/V Academician B. Konstantinov (ABK) and receiving ship (2) R/V Academician N. Andreyev (ANA) are spaced at 400 km. The ABK is moving through the five stations (N1–N5), keeping the same distance to the ANA during the experiments.
ity of signal propagation increases with frequency. Experiments have shown that acoustic pulses received at different distances varied in the shape of the pulse as it propagated in the waveguide. Pulse duration was reduced by a half or greater when the signal propagated in the waveguide over a distance of at least 3 km. To achieve complete synchronicity of arrival times of all the frequency components of the signal, a special type of frequency modulation would be necessary corresponding to the characteristic features of dispersion in the waveguide. Since the dispersion of the signal propagation speed depends nonlinearly on frequency, the frequency modulation should also be nonlinear to obtain the maximum compression of the signal. The limiting duration of the signal $\tau$ is in inverse relation to the effective frequency band $\Delta f$ of its spectrum $\tau \sim (\Delta f)^{-1}$. On the other hand, the signal duration $T$ of the pulse under the condition of its complete compression at a distance $L$ is determined by the frequency dispersion $\partial c/\partial f$ of the propagation speed $c$ and signal bandwidth $\Delta f$.

$$T = L \frac{\partial c}{c^2} \frac{\partial f}{\Delta f}$$

(3)

Thus, in the case of the signal compression due to waveguide dispersion, the signal intensity may increase by factor of

$$\frac{T}{\tau} = L \frac{\partial c}{c^2} \frac{\partial f}{\Delta f^3}$$

(4)

Hence, the effect of an increase in intensity is proportional to the distance traveled by the signal, the value of the waveguide dispersion, and the square of the signal frequency band. At the same time, an increase occurs in the signal-to-noise ratio by the recording equipment during the signal reception time. The compression of the signal while it propagates in an ocean waveguide leads to relative signal intensity gain $T/\tau$ that increases with signal propagation distance $L$ (Eq. 4). This can be most pronounced in the case of long-range propagation of a single mode, wide frequency band signal in an ocean waveguide.

**Conclusion**

Long-range ocean sounding may be based on the use of low frequency sound. Such waves may provide a favorable condition for development of nonlinear effects and the application of parametric arrays. These arrays differ from conventional arrays by their small size (transmitting aperture dimensions 1-3 m), a broad frequency band of the signal transmitted (50 Hz -1000 Hz), and a very sharp directivity pattern in the whole frequency range (2°- 3° in the main lobe, almost without side lobes). Our experiments demonstrated that a parametric array may provide the capability to investigate the crosswise array in an ocean flow that is very turbulent. Parametric arrays may be considered to be a promising tool for multi-frequency acoustical tomography techniques, especially for monitoring the range-dependent temperature and current distribution in a complex ocean environment using only one acoustical path and scintillation techniques.11

The International Science and Technology Center now supports the development of a full scale experimental model of a high-power parametric array for long distance ocean monitoring; specifically for the Fram Strait environment conditions. This project is ongoing in the N. Andreyev Acoustics Institute (Moscow, Russia) and in the Taganrog Technology Institution (Taganrog, Russia). The parametric array, developed using the principles of nonlinear acoustics, promises better experimental techniques for long-range multi-frequency acoustics experiments in complex ocean environments, when single mode transmission coupled with an ocean waveguide is needed.

**References**

Silentium Ltd. is a high-tech company specializing in developing innovative active acoustic technology and in providing the production tools associated with it. It aims towards confronting the increasing noise pollution generated from a wide range of electronic products. Silentium ANC utilizes unique cutting edge technology delivering up to 20dB(A) noise reduction across the entire audible spectrum and can be applied to versatile applications, such as server/networking equipment (IT), air treatment, and air conditioning machinery (HVAC) for the industrial, residential, automotive, as well as aviation markets, white goods, oven hoods, generator & car diesel engines, and more.

Silentium developed a generic controller for active acoustics and offers a range of tools for product developers and consultants to assist in applying active acoustics solution based on Silentium’s S-Cube™ controller. Silentium proposes two alternatives for implementing the electronic controller:

(i) S-Cube™ controller manufactured by Silentium.
(ii) RDK (Reference Design Kit), which is a set of instructions for adding the core components of the S-Cube™ to the customer’s electronic board.

Silentium recently introduced a new innovation & its Silentium Expert Program (SEP) program: Structural acoustics solution S-Barrier™

This ASAC technology is based on lightweight composite panels, equipped with piezoelectric actuators and a controller, producing counter vibrations superimposed with the structural vibrations, within the composite panel, to reduce the structure-borne noise.

Zone-to-Zone Complex sound field solutions: Silentium is currently in the process of designing

and developing a “smart” and robust system that can be positioned in an open space, for example: a car or an aircraft, reducing the disturbing noise independently from the sources of noise and without interfering with the conversations in the room. Basically, this kind of system is capable of producing quiet bubbles around the spatial zone of interest, for instance, around car or aircraft seats. Examples of ANC based products with Silentium’s active acoustics solution:

- Cray CX1 super computer incorporated Silentium’s noise cancellation system to reduce fan noise.
- ActiveSilencer™ Enclosure (ASE™) for Intel® Modular Server: Silentium has collaborated with Intel® to design the enclosure for reducing

Active Silencer™

S-Fan™

S-Cube™

Reference Design

S-Cube™ Development Kit

Active barrier

Silentium Expert Program (SEP):
Silentium established a new program to provide engineering companies and acoustic solution suppliers with the tools required for expanding their business opportunities by offering active acoustics solutions. Silentium experts will acquire the expertise to offer engineering support in the areas of ANC design, implementation and integration for OEM products & customized solutions. Silentium expert engineering team works closely with local field engineers to ensure OEM Partners success in developing and deploying their solutions based on products and technologies purchased from Silentium.
Igor B. Esipov is currently a Professor at Chair of Physics in the Gubkin University of Oil and Gas (Moscow, Russia). He has a part time position as head of the Nonlinear Acoustics Laboratory in the N. N. Andreyev Acoustics Institute. He is one of the scientists in Russia that has been concerned with the application of parametric arrays for long-range ocean acoustics. In 1990, under Esipov's supervision, researchers from the Applied Research Institute, N. Novgorod, performed a pioneering research experiment in parametric signal propagation in the Pacific Ocean using acoustical paths of up to 1000 km. He is a head of the Scientific Council on Acoustics of the Russian Academy of Science, a member of the board government of the Russian Acoustical Society, and a member of the Acoustical Society of America.

Konstantin Naugolnykh is a Senior Scientist at the National Oceanic and Atmospheric Administration, Earth System Research Laboratory and Zel Technologies, LLC. He received his Ph.D. in physics at the N. N. Andreev Acoustics Institute of the USSR Academy of Sciences in 1959. While at the Institute, he published papers concerning various fields of physical acoustics, including propagation of finite-amplitude acoustic waves and nonlinear acoustic phenomena. In 1963, he worked with R. Beyer and P. Westervelt in the Department of Physics at Brown University on parametric radiation studies. Dr. Naugolnykh, along with I. Esipov performed experiments on the use of the parametric array for long-range ocean sensing. His interests also were in radiation and propagation of an intense acoustic pulse that was generated by an electric discharge or a laser beam. These results were summarized in a monograph, Electric Discharges in Water (M., Nauka, 1971), written with N. Roy. He is a member of the Russian Acoustical Society and a fellow of the Acoustical Society of America.

Vladimir I. Timoshenko is a Professor and former head of Electro-Hydroacoustics and Medicine Technology in Taganrog Technology Institute (Taganrog, Russia). In 1965, at Taganrog, he founded a scientific school on Nonlinear Underwater Acoustics. Since then, nine Doctors of Science and about 50 Ph.D’s defended their thesis there. His original engineering research in nonlinear acoustic technology led to the development of a number of parametric arrays for sub-bottom profiling and shallow water research. Timoshenko created an art gallery and archeology museum at Taganrog where he also manages a series of regular concerts of musicians and choral groups. Vladimir Timoshenko was a winner of the State Prize on basic research in nonlinear acoustics (1985), and is Head of the South Branch of the Russian Acoustical Society. He is also a member of the French Acoustical Society.