ON THE FASCINATING PHENOMENON OF DIFFRACTION BY PERIODIC STRUCTURES

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Introduction

Although physically very complex, under a relatively wide range of conditions the diffraction of sound by a corrugated structure can be described and understood under the principle of a plane wave expansion. A plane wave expansion was first considered by Lord Rayleigh and was later applied by the team of Oswald Leroy while incorporating the mandatory mechanical coupling conditions between the two media separated by the corrugation. In essence the plane wave expansion (i.e., expansion into diffraction orders) is based on a Fourier series expansion with incorporation of propagation properties given by the dispersion relation and based on the acoustic wave equation. The fundamental connection between the periodic structure and the decomposition into different diffraction orders is the grating equation as also used in optics. Hence, it is not surprising that research teams in the 1980's and 1990's that worked in this area were also very experienced with Raman-Nath diffraction in acousto-optics. Besides curiosity in the area of physics, most researchers during the past century paid attention to diffraction of sound by corrugated structures because of the ability to transform bulk acoustic waves into surface acoustic waves. During that time special attention was paid to diffraction spectra and the appearance of anomalies. It was indeed found that certain anomalies (called Wood anomalies) coincided with the generation of Scholte-Stoneley surface acoustic waves. The latter are known to be difficult to generate, hence it was hoped corrugated surfaces would be more efficient. Scholte Stoneley waves propagate over large distances and are therefore of high interest for shallow water acoustic communication and also for high speed nondestructive testing of large surfaces and structures. Encouraged by the physics of sound being akin to that of light, Mack A. Breazeale's team[1] developed experiments to verify the existence of acoustic counterparts of diffraction effects in optics. One such effect was the Goos Hänchen effect[2] resulting in a backward displaced optical beam. The experiments lead to the observation of a similar effect in acoustics under the occurrence of diffraction, but could not be explained by the assumptions made at that time. It turned out later[3] that the use of inhomogeneous waves in combination with the plane wave expansion technique would be the key to explaining the effect and that the effect was due to a new type of surface acoustic wave.

> *Marcus Vitruvius Pollio (first century BC): "so by the arrangement of theaters in accordance with the science of harmony, the ancients increased the power of the voice."*

As scientific tools such as modern calculus and numerical methods that are applied today were not developed before the end of the 17th century, one must wonder if certain ancient phenomena could possibly be of even higher interest than modern day technology and experiences. It was pointed out by others[4–11] that Chichen Itza was known for the existence of a transformed echo at the base of the El Castillo pyramid (Kukulkan pyramid) that sounded pretty much like the chirp of a Quetzal Coatl. Indications based on prediction of Bragg scattering angles as a function of angle of incidence and frequency gave some insight but lacked actual correspondence with the experiments. As will be pointed out further Declercq *et al.*[12] made quantitative simulations taking into account mechanical coupling and interference at the origin of the diffracted waves and determined reflected features very much in agreement with experiments. An additional discovery found that not only was the staircase itself responsible for the specific feature of the echo, but also the signature of the incident sound.

Declercq,[12] while visiting Chichen Itza with fellow student Goffaux in 2003, discovered the raindrop acoustic effect in addition to the well-known chirp and this phenomenon was further studied experimentally and theoretically by Cruz and Declercq.[13]

Marcus Vitruvius Pollio, on the other hand, noted in the first century BC that the Greeks built their theatres following nature's footsteps: "they traced the voice as it rose, and carried out the ascent of the theater seats. By the rules of mathematics and the method of music, they sought to make the voices from the stage rise more clearly and sweetly to the spectators' ears. For just as organs which have bronze plates or horn sounding boards are brought to the clear sound of string instruments, so by the arrangement of theaters in accordance with the science of harmony, the ancients increased the power of the voice." New research revealed some interesting features as is described below.[14]

It turned out that diffraction of sound with incorporation of mechanical coupling and acoustic interference enables the understanding of both phenomena.

A recent re-emergence of interest in the field of diffraction of sound is connected with the development of phononic crystals.[15] It is known that such crystals exhibit very interesting filtering properties of interest in electronics and in seismology. Negative refraction for instance is found within this framework. Recently potential applications have been envisaged in which transformation of external bulk waves into phononic crystal propagation modes are considered. Although more complex than a corrugated surface, certain physical phenomena such as conversion into Scholte-Stoneley waves have been found already. Therefore one might expect continued interest in the diffraction phenomena at least for one more decade.

In what follows, a brief overview of a number of selected phenomena studied in the recent past with concise explanation and historical context will be presented.

Ultrasonic diffraction on periodic micro-structures

When ultrasound impinges a periodically corrugated material, it either scatters as it would on a regular surface, or it diffracts like light diffracts on a compact disk. Figure 1 shows an optical image of the cross section of a corrugated brass sample.

When frequencies are used resulting in wavelengths of the order of magnitude of the periodicity of the structure, it would diffract. However, as in Fig. 2, when a scan is made of such a sample at frequencies high enough to avoid diffraction, results are obtained showing good images of the corrugation.

If incident sound having a wavelength of the same order of magnitude as the corrugation periodicity is used, interesting diffraction effects occur. One such effect is the appearance of Wood anomalies in diffraction spectra of sound impinging the corrugated surface at normal incidence. In the 1980's and the 1990's such anomaly frequencies were used to generate Scholte-Stoneley waves on solids immersed in water.

In Fig. 3 experimental zero order reflection spectra are depicted, obtained using normally incident longitudinal waves impinging, from the waterside, a solid-water periodically corrugated surface.

In this framework one can seek optimization of surfaces to enhance surface wave stimulation.[18] Other anomalies also exist and they are, of course, not limited to normal incident sound. As a natural consequence, corrugated surfaces can be used to polarize ultrasonic waves.[19,20] They can also be used as sophisticated filters for complex frequencies (transient signals)[21,22] or to direct sound in particular directions in 3-dimensional space[23] when using 2-dimensional (doubly) corrugated surfaces.

Before we move on to a very interesting physical phenomenon it is important to mention that diffraction effects can also be exploited in air-coupled applications. A recent example is the measurement of the thickness of cylinders in a dry environment. Indeed Bragg scattering of sound on periodically stacked cylinders reveals, with high accuracy,[24] the cylinder diameter and is a rather easy technique to apply in industries such as steal-cord fabrication or even the pasta industry.

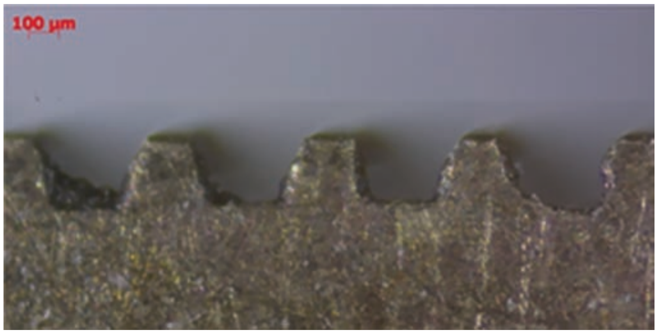

Fig. 1. Optical side view of a corrugated brass sample; Λ = 515 μm, h = 238 μm.[16]

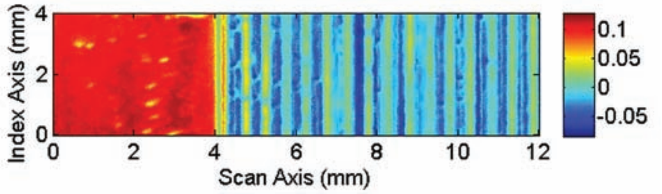

Fig. 2. An ultrasonic image of the sample presented in Fig. 1, based on a C-scan technique in which (in this example) the difference between the maximum amplitude of the plateau reflection and that of the valley reflection is represented by a color.[16]

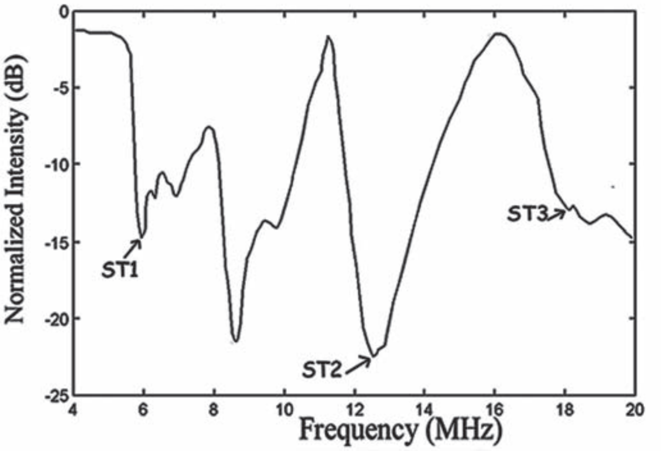

Fig. 3. Experimental normal reflection spectrum into water for a brass – water interface, Λ = 25 μm =, h = 66 μm. These results have been extracted from[17] *as a representative example of the research undertaken on this subject in the 1980's. The labels STn are added to show where anomalies appear that are related to the generation of Scholte-Stoneley waves.*

Perhaps a much more exciting phenomenon from a physical acoustics point of view is the backward displacement of beams when reflected off of a periodic structure. Breazeale and Torbett[1] performed experiments in the 1970's to determine whether there was an acoustic analogue of the optical Goos-Hänchen effect.

The Goos-Hänchen effect[2] predicts that light incident near the critical angle on a dielectric interface from an optically denser medium has a reflected beam that is laterally shifted from the position predicted by geometrical optics. The incident light beam transfers a portion of its energy into the optically rarer medium and excites an electromagnetic field that travels longitudinally for a certain distance along the interface. This energy is leaked back into the denser medium and interferes with the specularly reflected beam. This interference results in a reflected beam which exhibits a lateral displacement that appears as a forward beam shift. More complex structures such as multilayered media and periodically corrugated configurations of the optical grating type guide electromagnetic fields of the leaky-wave variety as well. The lateral displacement of a light beam reflected from a leaky-wave structure when a Gaussian light beam is incident upon it was studied by Tamir and Bertoni.[25] The Tamir and Bertoni[25] theory predicts that at a certain critical angle, a reflected beam shift may occur either in the forward or in the backward direction with respect to the incident beam. The early experiments of Schoch[26,27] using the acoustic analog of the Goos-Hänchen effect for an ultrasonic beam reflected from a liquid-solid interface showed a forward lateral displacement of the reflected ultrasound beam. Later, Breazeale and Torbett,[1] using a Schlieren photographic technique, observed a backward beam shift of a 6 MHz ultrasonic beam of 10 mm width, reflected from a superimposed periodic grating, confirming the backward beam displacement predicted by the theory of Tamir and Bertoni.[25] Although the backward shift was observed in acoustics, it was unclear which acoustic phenomenon caused it and above all there was no simulation method available at that time to model it. Much later Declercq *et al.* developed the necessary theoretical model based on a combination of the inhomogeneous wave theory and the plane wave expansion theory of diffraction to study the backward beam displacement.[3,28] The most important conclusions from that work were, first that the backward beam displacement is caused by a backward propagating Scholte-Stoneley wave generated by the incident beam. Such waves are known for their non-leaky feature when propagating on smooth surfaces, therefore they are used for long-distance nondestructive testing and for acoustic communication through sound propagating on the seafloor. What had not been known was that these waves become slightly leaky when propagating on corrugated surfaces. The developed theory described the leaky Scholte-Stoneley waves and enabled simulation of the backward displacement. Second, the theory also predicted that a bounded beam can still be displaced forward or backward at angles of incidence corresponding to leaky Rayleigh wave generation even though this had not been observed by Breazeale and Torbett.[1] The clue was that the beam must be sufficiently narrow in order to obtain the effect. Indeed, later experiments have confirmed the theory.[29] Backward beam displacement was considered an exotic phenomenon that only exists under very specific laboratory conditions. To verify these assumptions further research was performed and it turned out that practically every angle of incidence causes a backward displacement as long as the incident wave is an ultrasonic pulse. Herbison *et al.*[30] showed through an angular frequency spectrum analysis that practically every ultrasonic pulse contains frequencies that displace backward no matter what angle of incidence is used. It was also shown that the backward displacement does not just occur on the liquid side, as in the original experiments of Breazeale and Torbett,[1] but also on the solid side.[31] The experiments of Herbison *et al.*[30] were not based on the Schlieren technique enabling visualization of sound, but were based on quantitative measurements using a specially designed ultrasonic polar scan system. The technique allowed us to give the necessary evidence of the physical cause of the backward displacement, namely leaky Scholte-Stoneley waves. (see Fig. 4)

Influence of diffraction on the acoustic performance of theaters and auditoria

Periodic structures, currently very popular as phononic crystals, functioning as acoustic prisms and frequency selective mirrors hold promise for applications ranging from seismic wave deflection to accurate passive filters used in electronics. The Hellenistic theatre of Epidaurus, on the Peloponnese in Greece, which is well known for its extraordinary acoustic qualities, attracted Declercq's attention in the framework of his investigations in this field. The theater, renown for its extraordinary acoustics, is one of the best conserved of its kind in the world. It was used for music and poetry contests and theatrical performances.

Many assumptions existed concerning the reasons why the acoustics of this theatre were so extraordinary, yet not a single assumption was satisfactory. Declercq and Dekeyser[14] proposed that the acoustic quality of the theatre is due to the seat rows forming a corrugated structure. This research clearly showed and explained a filtering effect whereby the seat rows enhanced the acoustic quality of the theatre.[14] This study demonstrated

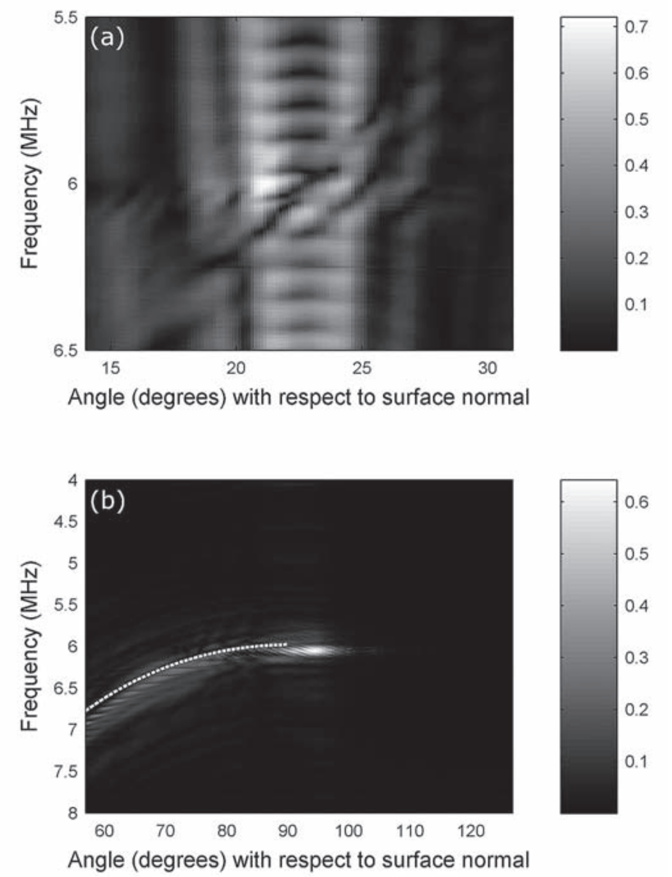

Fig.4. Angular spectrograms[30] confirming the results obtained by Breazeale and Torbett.[1] For a θ_i of 22.5º, a spectrogram from the region of the specularly reflected beam (a) shows backward shifted frequencies in the range 5.98-6.12 MHz. The complementary spectrogram from a scan to detect the backward surface wave (b) shows it occurring at 6.05 MHz. Propagating bulk modes also detected in (b) and shown theoretically by a dotted line. At other angles of incidence similar spectrum analysis shows that there is practically always some portion of sound backward displaced.

Fig. 5. The theater of Epidaurus in Greece well-known for its wonderful acoustic properties.

that an appropriate periodicity, in combination with properly selected building materials, has a significant influence on the theater's acoustics, and provides critically important design guidelines to improve the acoustical performance of modern outdoor theatres and sports stadiums. Investigations on the constructive influence of periodic seat rows, and later also on corrugated ceilings[32] as band pass filters on the acoustics of rooms and theatres, resulted in a new architectural paradigm that not only changed the way we look at ancient buildings but also how we design new buildings. (see Fig. 5)

Explanation of acoustic marvels at the Pyramid of Chichen Itza in Mexico

As pointed out earlier, perpendicular pulse-echo ultrasound experiments on corrugated surfaces were used in the 1990's to determine the corrugation periodicity or to generate surface waves at certain frequencies. More complicated periodic structures, such as photonic and phononic crystals, were developed in the late 1990's and into the 21st century. The first element of added complexity appeared in the 1990's in experiments on oblique pulse-echo ultrasound. These experiments actually illuminated a striking feature of the El Castillo pyramid in Chichen Itza, on Mexico's Yucatan peninsula. There, acoustic waves generated by a handclap were back-reflected by the immense ziggurat causing them to sound not like a handclap, but like a chirping Quetzal. This phenomenon has long intrigued not only tourists and archeologists, but also acousticians.[4–11] Declercq *et al.*[12] were the first to deliver a full explanation of this phenomenon based on a theoretical model and subsequent numerical simulations. This study received wide attention because of the widespread interest in this phenomenon in other branches of science and culture and because of the possibility that Mayans actually built pyramids not only as tremendous calendars but also as recordings of the chirp of the holy Quetzal Coatl (Kukulkan).

Another phenomenon, the so called raindrop effect, earlier believed to be caused by the partial hollowness of the pyramid, was explained by Cruz and Declercq[13] as a natural effect caused by the staircase. (see Fig. 6)

Conclusions and continued research

Without being exhaustive we have tried to give a brief overview of fields in which diffraction of sound by a periodic structure is of importance and we have to some extent explained our share in this exciting research area. As the fabrication of new micron and nano materials has become common practice, it is believed that more problems and applications involving sound diffraction will need study the coming years. New results and new physical phenomena related to diffraction of sound are currently under investigation and can be expected as publications in the near future. **AT**

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Fig. 6. The El Castillo pyramid of Chichen Itza (photo taken by the author).

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Nico Declercq and his daughter at Chichen Itza

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