The Acoustics of Woodwind Musical Instruments

The oldest known instrument family produces a wide range of tone colors and pitch using a range of interesting physics.

Introduction and Overview

Woodwind instruments have been played by humans, possibly including Neanderthals, for more than 40,000 years. Tubular pieces of bone, pierced with holes, and similarly shaped fragments carved from mammoth ivory have been discovered in caves in Europe; they are flutes, perhaps end-blown like a Japanese shakuhachi (Atema, 2014). Artificial musical instruments may be even older, but those made of perishable materials, including possibly drums, lyres, and didgeridoos, cannot compete archaeologically.

Woodwind instruments are classified by the sound source: air-jet instruments (e.g., flutes; Figures 1a and 2), single-reed instruments (clarinet, saxophone; Figures 1a and 3), or double reeds (oboe, bassoon; Figure 1a). Because of their diversity, the woodwind family provides a wide range of timbre, and orchestral composers often contrast their tone colors by passing a theme between different woodwind instruments (see examples at www.phys.unsw.edu.au/jw/AT). Beyond orchestras, woodwinds from folk traditions extend the timbre range further. Organ pipes (Angster et al., 2017) are also arguably woodwinds; they are excited either by an air jet or a single brass reed.

Like brass instruments (Moore, 2016), woodwind instruments convert energy in the steady flow of high-pressure air from the lungs into that of oscillations and sound waves. Also like brass, woodwinds have a sound source whose properties are strongly nonlinear. For brass, the source is the player’s lips; for woodwinds, it is either an air jet or one or a pair of reeds. This source interacts with a resonator (largely linear), which is usually an acoustic duct, called the bore. In woodwinds, the effective length of the duct is varied when tone holes in its wall are opened or closed by the player. Usually, the instrument plays at a frequency near one of the duct resonances. Under the player’s control, interactions between source and resonator determine pitch, loudness, and timbre.

The player’s mouth provides air with a pressure of typically a few kilopascals and a flow rate of tenths of a liter per second, both varying with instrument, loudness, and pitch; this gives an input power of ~0.1 to a few watts. Oscillations of the air jet or reed modulate airflow into the bore, where resonant standing waves in turn produce fluctuating flow or pressure at the mouthpiece. These, in turn, control the input, a process called auto-oscillation discussed in Sound Production with an Air Jet and Sound Production with Reeds. Only a small fraction (order of 1%) of the input energy is radiated as sound from any open tone holes and from the remote end. Most of the input energy is lost as heat to the walls. This inefficiency is fine, however; players typically input a few hundred milliwatts, and an output of even a few milliwatts can produce about 90 dB at the player’s ears. Most woodwinds are not as loud as brass. However, the piccolo, whose playing range includes the ear’s most sensitive range, is readily heard above the orchestra.
The Bore, Its Resonances, and Its Impedance Spectrum

The ducts of the larger clarinets, saxophones, and bassoon are bent to make tone holes and keys easier for the player to reach. These bends have only a modest acoustical effect. For most flutes and the clarinet, much of the duct is approximately cylindrical, whereas for the oboe, saxophone, and bassoon, the duct is nearly conical.

Let’s begin by considering a completely cylindrical bore (Figure 4), open to the air at both ends, a case approximated by the flute (Figure 1a) or the hybrid instrument in Figure 5, bottom. Inside the bore, the acoustic pressure can vary significantly, positive or negative. At the open ends, however, the acoustic pressure is small; the total pressure is close to atmospheric. So, if we look for resonant modes of oscillation in the bore, the boundary condition at both ends is a node for pressure and freedom for large flows in and out.

The mode diagrams in Figure 4 plot acoustic pressure (red) and acoustic flow (blue) against position along the bore. The diagram in Figure 4, top left, shows that half a sine wave fits the bore with a pressure node near each end, allowing a lowest mode whose wavelength ($\lambda$) is roughly twice the length ($L$) of the pipe, say $\lambda \approx 2L$.

From its open embouchure hole to the other open end, the nearly cylindrical flute in Figure 1a has a length of 0.63 m. The frequency $f_1 = c/\lambda \approx c/2L = 270$ Hz, where $c$ is the speed of sound. This is a little higher than its lowest note, B3, at 247 Hz, played with all the tone holes closed. The difference should not surprise us because the instrument is neither exactly cylindrical nor completely open at the mouthpiece end.

Considering the five mode diagrams at Figure 4, left, we see that the zero-pressure boundary conditions near the ends enclose, respectively, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, and $\frac{5}{2}$ wavelengths. Using $n$ for the mode number, the wavelengths and frequencies are thus approximately

$$\lambda_n = 2L/n \quad \text{so} \quad f_n = c/\lambda_n = nc/2L \quad \text{so} \quad f_n = nf_1 \quad (1)$$

Frequencies in the ratio 1:2:3, and so on, make the harmonic series. With a flute whose lowest note is C4 (262 Hz), a player can change the air-jet speed and length, thus exciting the bore to vibrate at frequencies $nf_1$, producing the notes with $f_1$ (C4), $2f_1$ (C5), $3f_1$ (G5), $4f_1$ (C6), $5f_1$ (E6), $6f_1$ (G6), and $7f_1$ (a note between A6 and A#6), all without moving the fingers (see sound files and video at www.phys.unsw.edu.au/jw/AT).

A further important point is that a nonsinusoidal periodic sound, with period $T = 1/f_1$, contains harmonics with frequencies $nf_1$. So, for a low note on the instrument operating at $f_1$, the resonances at $2f_1$, $3f_1$, and possibly higher multiples help radiate power at these upper harmonics of the sound and contribute to making the timbre brighter.

A clarinet is also roughly cylindrical but, unlike a flute, it is almost completely closed at the mouthpiece by the reed (Figure 1c). Figure 4, right, shows the modes; this bore can accommodate $1/4$, $3/4$, $5/4$, and so on, wavelengths

$$\lambda_n = 4L/(2n - 1) \quad \text{so} \quad f_n = c/\lambda_n = (2n - 1)c/4L \quad \text{so} \quad f_n = (2n - 1)f_1 \quad (2)$$

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Woodwind Acoustics

So, with the same length, the \( f_1 \) of an ideal closed-open, cylindrical tube is half that of the open-open tube so it plays an octave lower. Because of its bell and a flare leading to it, the clarinet does not play a full octave lower than the flute. The clarinet’s lowest note is D3 or C#3 (139 Hz) compared with C4 or B3 (247 Hz) for the flute. The available higher modes for an ideal closed-open cylinder have frequencies 3, 5, or 7 times that of the lowest. One consequence is that, for the low notes on a clarinet, the first few even harmonics are substantially weaker than the adjacent odd harmonics.

The different conditions at the mouthpiece have another important consequence. A flutist can play an octave of notes using the first resonance and different effective lengths for each note. Then, because its resonances have \( f_2 = 2f_1 \), the flutist can play notes in a second octave using mainly the same fingerings but producing a faster jet. For the clarinet, the second mode has \( f_2 = 3f_1 \), so the clarinetist is on the 12th note of a diatonic scale when similar fingerings can be reused.

The bores of the oboe, bassoon, and saxophone are not cylindrical but are mainly conical. For axial waves in a cone, the cross section varies as one over the square of the distance from the apex, comparable with isotropic, spherical radiation. Consequently, the standing waves are not simple...
sine and cosine functions as in Figure 4. For a complete, open cone of length \( L \), the solutions to the wave equation can form a complete harmonic series, with \( nc/2L = f_n = nf_1 \). These are the same frequencies as in an open-open cylinder of the same effective length. Of course, no instrument is a complete cone; that would leave nowhere for air to enter. Simply truncating a cone gives resonances that are inharmonic. However, harmonicity is approximately restored if the truncated apex is replaced by a mouthpiece having an equal effective volume. (That effective volume includes an extra volume representing the compliance of the reed.)

**Acoustic Impedance**

The acoustic response of the instrument bore is quantified by its acoustic impedance spectrum, \( Z(f) \), the acoustic pressure at the mouthpiece divided by the acoustic current into the mouthpiece and measured in acoustic megohms or MPa-s-m\(^{-3} \). Figure 6 shows the magnitude of the measured impedance spectra \( Z(f) \) for 5 ducts having the same effective length of about 33 cm. In the spectrum for the cylinder (Figure 6, bottom), the minima in \( Z \) (largest flow for given pressure) correspond to the modes of the open-open pipe and the maxima to those of a closed-open pipe. The minima form a complete harmonic series with \( f_1 \) near 520 Hz or about C5 (cf. Eq. 1), and the maxima form a series with \( f_1 \) near 260 Hz (C4) and its odd multiples (Eq. 2).

Above that of the cylinder is the measurement for a flutefingered to play C5. In the downstream half of the instrument, nearly all the tone holes are open, giving it an effective length corresponding approximately to its closed upstream half, as suggested by the schematic. The impedance minimum corresponding to C5 is circled.

At low frequencies, \( Z(f) \) for both the flute and clarinet (see Figure 6) resemble that of a simple cylinder; for these frequencies, the bore is effectively terminated near the first open holes. The clarinet, however, is almost closed at the mouthpiece by its reed so that it operates at maxima in impedance, and with a similar closed length of bore, it plays C4, an octave lower than the flute.

In Figure 6, top, is the measurement of a truncated cone, with a cylinder of equal volume replacing the truncation. Below is the measurement of a soprano saxophone, fingered to play C5 (trill fingering), with the fundamental impedance maximum circled. As for the clarinet, the reed requires large pressure variation for small flow and so plays at impedance maxima. The varied high-frequency behavior is discussed in **Tone Holes, Register Holes, End Effects, and the Cutoff Frequency**.

Returning to \( Z(f) \) for the cylinder, it is worth considering the time domain. Suppose we inject a short pulse of high-pressure air at the input. It travels to the open end where, with the constraint of negligible acoustic pressure, it is reflected with a pressure phase change of \( \pi \) so that it becomes a pulse of negative pressure. After one round-trip of the 33-cm

![Figure 6. Semilog plots of measured amplitudes of acoustical impedance spectra (after Wolfe et al., 2010). The flute (second from bottom) and saxophone (second from top) have fingerings that play C5 (523 Hz) and the clarinet (middle) plays C4 (262 Hz). In all cases, this means tone holes open in the bottom half of the instrument, as indicated in the schematics. The length of the cylinder (bottom) was chosen to put its first maximum at C4 and its first minimum near C5. The cylinder + cone has a first maximum at C5 (top). Thus, all five ducts have the same acoustical length (~L). In the flute, the disappearance of resonances around 4 kHz is an interesting effect that limits the range of the instrument. The small volume of air in the “dead end” beyond the embouchure hole (see Figure 1b) and the air in that hole constitute, respectively, the “spring” and mass of a Helmholtz resonator, which helps the instrument play in tune. At resonance, however, this short circuits the bore.](image-url)
cylinder (about $2L/c \sim 2$ ms), it returns to the input where this now negative pressure pulse makes it easy to inject the next pulse of air, whether it comes from a device to measure $Z$ or from the jet of a flute. So, a standing wave with a period of $\sim 2$ ms or a frequency of $\sim 500$ Hz produces a minimum $Z$ and it can drive a jet oscillating near that frequency.

But what happens instead if that returning negative pulse meets an input nearly closed by a clarinet reed? The negative pressure pulls the reed more closed, and the pressure pulse is reflected this time with no phase change. So, on its second round-trip, a negative pulse now travels down the bore where it is reflected at the open end with a $\pi$ phase change and returns as a positive pulse. This time, it can push the reed open, let more air in, and thus amplify the next injected pulse. Thus, after two round-trips (about $4L/c \sim 4$ ms), a positive pulse returns and the clarinet cycle repeats. An oscillation with frequency $c/4L \sim 250$ Hz sees $\sim 4$ ms), a positive pulse returns and the clarinet

To aid the production of the second register, a register hole is often used. For example, in the recorder (Figure 2a), the thumbhole is half-covered to provide a leak at a position where the first mode of oscillation would normally have substantial pressure. This weakens (and detunes) the first resonance and thus allows the second resonance to determine the pitch; the instrument plays its second register. (The frequency dependence of the inertial effect of air near tone holes means that, in a simple cylindrical bore, the same fingering would play somewhat less than an octave between two registers because the higher note would have a longer end effect. For this and other reasons, real instruments depart from cylindrical shape to improve intonation.)

At a sufficiently high frequency, the inertia of nearby air virtually seals the tone holes. So, above a value called the cutoff frequency ($f_c$), the standing wave is little affected by open tone holes. For the clarinet in Figure 6, middle, $f_c \sim 1.5$ kHz. Below $f_c$, the maxima (or minima) are spaced about 500 Hz apart and open tone holes determine the effective length of the bore. Above $f_c$, their spacing is roughly half this frequency because now the effective length is almost the entire bore, despite many open tone holes.

The cutoff frequency therefore limits the range of harmonic extrema in $Z(f)$ and that limits which high harmonics are radiated efficiently. From baroque to romantic to modern instruments, more and successively larger holes raised $f_c$, contributing to increased power in higher harmonics and thus to increased timbral brightness and loudness.

From 1831 to 1847, Theobald Boehm revolutionized the flute. In his system, a dedicated tone hole opened for each ascending semitone. This meant that, for most fingerings, open holes were more closely spaced. The tone holes themselves were also larger, which required keys with pads to close them. A system of axles and clutches allowed the keys to be operated by eight fingers and one thumb and made playing all the keys relatively easy. The modern oboe, clarinet, and saxophone use some of his ideas.

Woodwinds have from thousands to millions of possible fingerings, of which only a small fraction are regularly used.
Sound Production with an Air Jet

The directional instability of a jet is demonstrated by a rising plume of cigarette smoke in still air. A jet deflects alternately in lateral directions and, after a while, sheds vortices. In the air-jet family, a narrow, high-speed jet is blown across a hole in the instrument toward a fairly sharp edge, the labium. In flutes, in the end-blown shakuhachi, and in panpipes, a high-speed jet emerges from between the player’s lips. In the recorder and ocarina, a tiny duct called a windway guides the jet to the labium, making it easier to sound a note.

At the labium, downward deflections of the jet flow into the bore and upward deflections flow outside the instrument (Figure 7). With a suitable phase, standing waves with large flow amplitudes at the embouchure hole can entrap the jet in a feedback loop that causes it to be directed alternately into and outside the bore, thus maintaining the amplitude of standing waves in the bore and, in the starting transient, increasing it. The frequency of spontaneous deflections of the flute jet increases with jet speed, so successively faster jet speeds (typically tens of meters per second) excite successively higher resonances via a mechanism involving several subtleties (Fletcher and Rossing, 1998; Auvray et al., 2014). The different possibilities of lip aperture and jet speed, angle, height, and length give the player a range of parameters to control pitch, loudness, and timbre. For example, the pitch can be lowered significantly by rolling the instrument so that the lower lip occludes more of the embouchure hole, which increases the end effect. Turbulence produces a broadband signal (we can hardly call it “noise” in this context), which is an important part of timbre, especially for panpipes.

Sound Production with Reeds

In reed instruments, one or a pair of reeds are deflected by the varying pressure in the bore so as to modulate the airflow into the instrument. For clarinets and saxophones, the single reed is fixed on a mouthpiece (Figure 8) and bends like a cantilever to produce an oscillating aperture. Double-reed instruments (e.g., oboe and bassoon) have two symmetrical and curved blades that alternatingly flatten and curve to close and open the aperture (Figure 1, d and e). Clarinet and saxophone reeds are damped by the player’s lower lip; the double reeds of the oboe and bassoon by both lips.

Consider the clarinet mouthpiece in Figure 8. Starting from zero, the blowing pressure \( P \) is gradually increased and the steady flow \( U \) past the reed is recorded. For now, neglect standing waves inside the mouthpiece. Initially, the airflow increases rapidly with increasing \( P \); if all the air’s kinetic energy is dissipated in downstream turbulence, we expect, from Bernoulli’s equation, \( U \propto \sqrt{P} \). At large \( P \), however, the pressure closes the reed against the mouthpiece and the flow must go to zero and does so at lower \( P \) if we increase the lip force (Figure 8, red curve). Consider a point on the right side of the curve, where \( U \) decreases with increasing \( P \). For steady flow, the ratio \( P/U \) is positive (inverse slope of dashed line); the mouthpiece is like an acoustic resistance, taking power out of the incoming high-pressure air. But for small acoustic signals, \( \partial P/\partial U \) (solid tangent) is negative; the mouthpiece is a negative resistance, inputting acoustic power to the clarinet. At the peak of the curves in Figure 6, the bore impedance is resistive, and if its resistance is larger than \(-\partial P/\partial U\), then the reed
inputs more acoustic power than the bore loses. The result is that a small signal at the fundamental frequency increases exponentially until the small signal approximation is no longer valid (Li et al., 2016). (This argument treats the inertia of the reed as negligible, which ceases to be even approximately true for high notes.) This simple model also explains the final transient. For example, if the reed is abruptly stopped by the tongue, the fastest possible decay rate is determined by the quality factor of the operating resonance.

Qualitatively, the double reeds of oboe, bassoon, and others share the same principles, although the geometry of the reed and its motion are both more complicated. The compliance and inertia of the reeds, the acoustic resistance in the narrow passage between them, and the Bernoulli force on the reed play larger roles, and the difference between the quasi-static and oscillating regimes is greater (Almeida et al., 2007).

Controlling the Output Sound

In Sound Production with an Air Jet, I mentioned some of the jet control parameters used by flutists. The argument immediately above shows the importance of blowing pressure and lip force(s) for reed players. Complicating life for players is that control parameters often affect several output properties. For example, blowing pressure and lip force both affect each of loudness, pitch, and spectrum.

Another aspect of control involves the player’s vocal tract. A vibrating reed produces acoustic waves in both directions, and the acoustic force acting on the reed is approximately proportional to the series combination of the impedances of the bore and vocal tract. Especially at high frequencies, where the impedance peaks of the bore are weaker (Figure 6), resonances in the tract can affect pitch and timbre or control multiphonics. Tuning the tract resonances is necessary for playing the high range of the saxophone and for the famous clarinet portamento that begins Gershwin’s Rhapsody in Blue (Chen et al., 2008, 2009).

The diversity and complexity of woodwinds and the range of interesting physical effects involved continue to engage the attention of researchers. Excellent technical treatments of woodwind acoustics are given by Nederveen (1998), Fletcher and Rossing (1998), and Chaigne and Kergomard (2016). My lab concentrates on the musician-instrument interaction (reviewed by Wolfe et al., 2015) and provides introductions with sound files and video (Music Acoustics, 1997).

References


Biosketch

Joe Wolfe is a professor of physics at the University of New South Wales (UNSW) Sydney, where he leads a lab researching the acoustics of the voice and musical instruments, especially woodwinds, brass, and didgeridoo. In the past, he has worked at Cornell University, Ithaca, NY, at CSIRO (Australia’s national research organization), and the École Normale Supérieure, Paris. He has received national and international awards for research and teaching. Outside of physics, he plays double reeds and the saxophone. His trumpet concerto has been performed several times, and his quartet has had concert performances on all continents except Antarctica.