James Clerk Maxwell and the Physics of Sound

Philip L. Marston

Postal:
Physics and Astronomy Department
Washington State University
Pullman, Washington 99164-2814
USA

Email:
marston@wsu.edu

The 19th century innovator of electromagnetic theory and gas kinetic theory was more involved in acoustics than is often assumed.

Introduction

The International Year of Light in 2015 served in part to commemorate James Clerk Maxwell's mathematical formulation of the electromagnetic wave theory of light published in 1865 (Marston, 2016). Maxwell, however, is also remembered for a wide range of other contributions to physics and engineering including, though not limited to, areas such as the kinetic theory of gases, the theory of color perception, thermodynamic relations, Maxwell's "demon" (associated with the mathematical theory of information), photoelasticity, elastic stress functions and reciprocity theorems, and electrical standards and measurement methods (Flood et al., 2014). Consequently, any involvement of Maxwell in acoustics may appear to be unworthy of consideration. This survey is offered to help overturn that perspective.

For the present author, the idea of examining Maxwell's involvement in acoustics arose when reading a review concerned with the propagation of sound waves in gases at low pressures (Greenspan, 1965). Writing at a time when he served as an Acoustical Society of America (ASA) officer, Greenspan was well aware of the importance of the fully developed kinetic theory of gases for understanding sound propagation in low-pressure gases; the average time between the collision of gas molecules introduces a timescale relevant to high-frequency propagation. However, Greenspan went out of his way to mention an addendum at the end of an obscure paper communicated by Maxwell (Preston, 1877). There, Maxwell examined the relationship between kinetic theory and recent measurements of the speed of sound in mercury vapor (Kundt and Warburg, 1876). The context and importance of these developments are clarified in the present article.

Also relevant to Maxwell's involvement with, and influence in, acoustics was his stature in mathematical physics during his short lifetime (1831-1879). Maxwell's stature resulted in a secondary involvement in acoustics, his services as a peer reviewer of several important acoustics manuscripts, examples of which are discussed here. Another secondary influence was through his teaching and writing. For example, Horace Lamb (1849-1934) studied Maxwell's highly mathematical *Treatise on Electricity and Magnetism* (Maxwell, 1873a) and was taught by Maxwell at Cambridge in the 1870s. Of Maxwell's professional interactions, however, those with Lord Rayleigh (1842-1919) are emphasized here.

Resources and Chronological Summary

Any endeavor to examine Maxwell's life and thought is aided by the availability of four resources: (1) the early biography (Campbell and Garnett, 1882) commissioned by Maxwell's widow and designated here as the *Life*; (2) a compilation of many of Maxwell's scientific papers (Niven, 1890); (3) compilations of Maxwell's

Figure 1. *Maxwell's portrait from the 1882 printing of the* Life *scanned from a copy owned by the present author.*

correspondence and publications pertaining to molecules, gases, and related issues along with summaries of selected subsequent developments (Garber et al., 1986, 1995); and (4) partial compilations of Maxwell's scientific correspondence and additional publications and manuscripts (Harman, 1990, 1995, 2002). The chronology of Maxwell's life and some relevant developments may be summarized as follows:

- 1831 (June 13): born in Edinburgh, Scotland
- 1847–1850: studies at the University of Edinburgh
- 1850–1856: enrolls in Cambridge University and remains there after his first degree in 1854
- 1856–1860: holds the chair in Natural Philosophy, Marischal College, Aberdeen, Scotland
- 1858: weds Katherine Mary Dewar; they have no children
- 1860–1865: holds the professorship in Natural Philosophy, King's College London
- 1865–1871: retires from King's College London, at least partially for reasons of health, and typically resides at his rural home in Glenlair, Scotland
- 1871–1879: holds the professorship in Experimental Physics at Cambridge
- 1879 (November 5): dies of painful stomach cancer
- 1879 (December)–1884: Rayleigh holds the professorship in Experimental Physics, Cambridge

Electromagnetic Theory and Waves

Maxwell's progression of research resulting in the electromagnetic wave theory of light may be summarized by his sequence of publications and insight provided from his correspondence (Marston, 2016). By 1855 and 1856, in his early papers, Maxwell had explored the usefulness of "physical analogies" between the motion of an incompressible fluid and Faraday's electrical lines of force. He also showed how Faraday's electromagnetic induction of currents and electric fields (**E**) could be expressed using a function he called by 1873 the vector potential (**A**). Expressed using modern notation, he found $\mathbf{E} = -\partial \mathbf{A}/\partial t$, where t denotes time.

By 1856, Maxwell had also developed a geometrical method for constructing diagrams of field lines. His series of papers in 1861 and 1862 developed an analogy of electrical vortices and electric particles for considering the coupled dynamics of electromagnetic fields, leading to his initial prediction of electromagnetic waves. These papers also introduced the notion of electromagnetic stresses (the Maxwell stress tensor) and of displacement currents. In his more famous paper (Maxwell, 1865), he derived the wave equation for the magnetic intensity (**H**) through the hypothesis of displacement currents without directly relying on a vortex analogy. In these papers, the predicted velocity of electromagnetic waves (hypothesized to correspond to light waves) depended on the ratio of certain electromagnetic quantities in absolute units. In the mid-1860s, Maxwell and associates made a significant effort to improve the accuracy of electrical measurements. One outcome was a new measurement of relevant electromagnetic quantities in 1868, giving additional support to Maxwell's electromagnetic theory of light. (By the mid-1880s, Rayleigh had located a systematic error in related properties of a standard for electrical resistance that improved the agreement with Maxwell's theory of light [Strutt, 1924].) Maxwell examined in his *Treatise* (1873a) the measurement of electrical quantities and their physical significance along with a reformulation of electromagnetic theory. Two of the important results for the history of physics were Maxwell's diagram of the spatial relationship of electromagnetic fields in a propagation wave (**Figure 2**) and his prediction and analysis of optical radiation pressure.

In Article 830 of his *Treatise*, Maxwell expressed his confidence in the underlying assumptions of his electromagnetic theory of light. The *Treatise*, however, was difficult for students and Maxwell's contemporaries to understand, in part because of Maxwell's use of notation associated with quater-

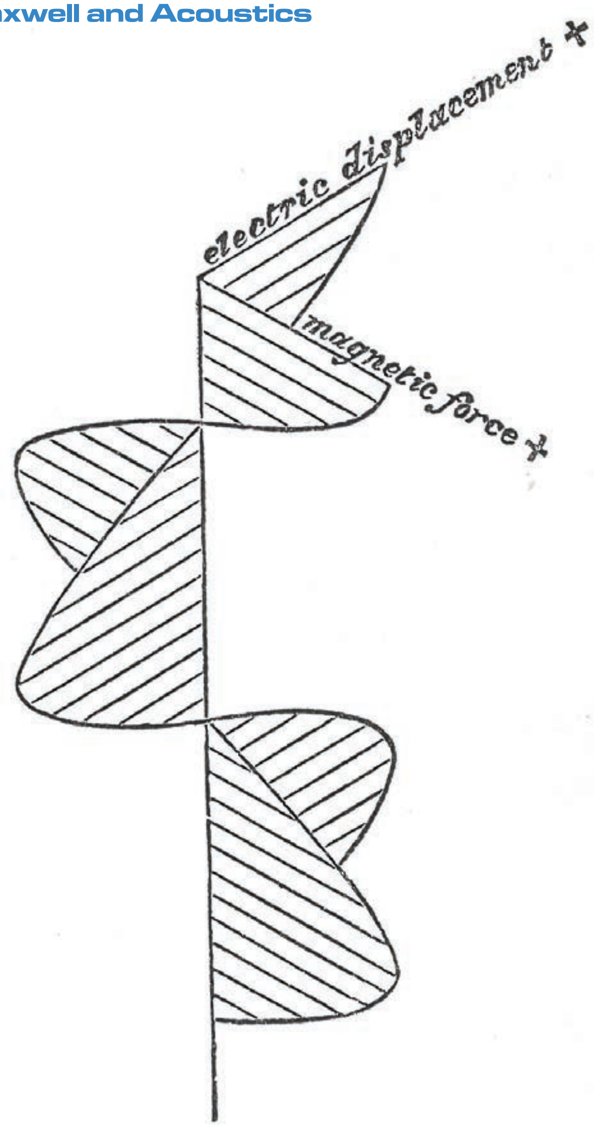

Figure 2. Maxwell's diagram from his 1873 Treatise *on the spatial relationship of the electromagnetic fields "at a given instant" for an electromagnetic wave. This image was scanned from an 1892 printing.*

nions. (As with his other writings, his *Treatise* predated efficient vector and tensor notation.) The *Treatise* was unique in its liberal display of quantitative electric and magnetic field diagrams and its presentation of spherical harmonic functions. Heinrich Hertz (1857-1894) provided major support for Maxwell's electromagnetic theory of waves through his experiments published in 1888 (Hertz, 1893). Hertz also contributed to what has been described as a purification of Maxwell's theory (Sommerfeld, 1952). Sommerfeld recalled how the combined formulations of Maxwell, Hertz, and Heaviside systematized the totality of electromagnetic phenomena in the early 1890s.

Maxwell's First Kinetic Theory of Gases, 1859, 1860

Between the early stages of his development of electromagnetic theory, Maxwell initiated a quantitative formulation of the kinetic theory of gases by introducing probabilistic distributions into physical theory. He also provided a physical basis for modeling transport coefficients (Garber et al., 1986). The principal publication (Maxwell, 1860) expanded on his presentation at the 1859 meeting of the British Association for the Advancement of Science (BAAS). Maxwell modeled a gas as if it consisted of a large number of hard elastic spheres that interacted only during collisions. In the absence of flow of the gas, Maxwell postulated that after the collision of spheres, the three Cartesian components of velocity were independent. This gave identical probability distributions for each component and the following number differential of spheres (per unit volume; dN) having velocity magnitudes between v and $v + dv$

$$\mathrm{d}N = 4N\,\alpha^{-3}\pi^{-1/2}v^2\exp(-v^2/\alpha^2)\,\mathrm{d}v \qquad (1)$$

where N is the total number of spheres per unit volume and α is a constant related to the average velocity ($\langle v\rangle = 2\alpha\pi^{-1/2}$) and mean squared velocity [$\langle v^2\rangle = (3/2)\,\alpha^2$]. By calculating the pressure (P) resulting from collisions with the side of the vessel and equating that pressure with the gas law of "Boyle and Mariotte," $P = K\rho$, where ρ is the density of the gas and K is proportional to the absolute temperature (T in modern notation), Maxwell concluded that $\alpha^2 = 2K$. Here, K is proportional to T/M, where M is the mass of each sphere. He also concluded that for a given P and T, $NM\langle v^2\rangle$ is the same for all gases. By that stage of his paper, Maxwell had separately concluded that $(M/2)\langle v^2\rangle$ is the same for differing sets of spheres in equilibrium (an energy equipartition theorem) and hence that his model explained the observed behavior of gases. Using modern terminology, **Equation 1** is an example of a Maxwell-Boltzmann probability distribution.

Some of the other quantities examined by Maxwell depend on the size of the colliding gas particles through (in his notation) s, the sum of the radii of the colliding spheres. For simplicity in what follows, attention is restricted to the situation of identical spheres. Maxwell found that the "mean path of each particle" (L) between collisions was $L = 1/(Ns^2\,\pi 2^{1/2})$.

Throughout his analysis, he assumed that the molecular size was sufficiently small so that $s \ll L$ and $s \ll N^{-1/3}$, the mean molecular spacing. He proceeded to calculate the transport properties having direct relevance to acoustical phenomena. He found the shear viscosity (μ) to be $\mu = \rho L\langle v\rangle/3 = (M\langle v\rangle/s^2)/(3\pi\sqrt{2})$, which does not directly depend on the P of the gas. (He noted that lack of dependence on P with some surprise, first in a letter to G. G. Stokes in May 1859.) This required considering collisions between molecules from adjacent gas regions having different flow velocities. He also obtained predictions for thermal conductivity and, for mixtures, gas mass diffusivity. (By 1862, Rudolf Clausius published a significant criticism of the result for thermal conductivity, quickly appreciated by Maxwell as a proper concern.) More relevant to the physics of sound, however, was Maxwell's prediction for the ratio of specific heats at a constant pressure and volume (c_p and c_v, respectively) for the gas

$$\gamma = c_p/c_v = 1 + 2/(3\beta) \tag{2}$$

where for the spherical gas particles in thermal equilibrium $(\beta - 1)$ = (rotational energy)/(translational energy). For rough spheres, from Maxwell's equipartition of the energy principle, he found $\beta = 2$ and $\gamma = 4/3$. He noted, however, that for air, γ had been measured to be approximately 1.408, a result that he took to be "decisive against the unqualified acceptation of the hypothesis that gases are such a system of hard elastic particles." By 1860, Maxwell was certainly aware that Laplace's assumption of negligible heat flow during sound propagation in gases results in the speed of sound at audible frequencies of $c = \sqrt{(\gamma P/\rho)}$. He included that relationship a decade later in his general textbook *Theory of Heat* (Maxwell, 1871).

Maxwell's Confirmation of the Pressure Independence of Viscosity, 1866

To investigate the predicted independence of μ on pressure, Maxwell developed a new method of measuring viscosities by measuring the damping rate of rotational oscillations of layers of disks near stationary plates separated by gas. These experiments on the friction of gases were done in his residence in London with assistance from his wife. The measurements confirmed the predicted independence on pressure (Maxwell, 1866).

Maxwell's Improved Kinetic Theory, 1867

Maxwell (1867) reformulated kinetic theory by considering the consequences of binary molecular encounters, providing a rigorous background for subsequent advance. He gave a new derivation of the velocity distribution function now denoted by $f(\mathbf{v})$, giving a result equivalent to **Equation 1**. The condition $f(\mathbf{v}_1)\,f(\mathbf{v}_2) = f(\mathbf{v}_{f1})\,f(\mathbf{v}_{f2})$ produced equilibrium, where $\mathbf{v}$ and $\mathbf{v}_f$ correspond to the initial and final velocity vectors of molecules 1 and 2, respectively, associated with the collision. He also used the conservation of translational kinetic energy in that derivation. Of greater importance, however, was his new approach to calculating transport coefficients through the introduction of the following transfer equation for the rate of change in a quantity of interest

$$(\delta Q/\delta t)dN_1 = (Q_f - Q)Vb\,db\,d\phi\,dN_2\,dN_1 \tag{3}$$

where the quantities of interest (Q) are functions of the Cartesian components of velocity referred to a coordinate system connecting the molecular centers of force at the distance of closest approach, ϕ is the azimuthal angle, b is the impact offset radius associated with the collision, and V is the relative velocity. From the laws of mechanics, Maxwell expressed the scattering angle after each collision in terms of an integration involving b, V, the molecular masses, and the repulsive force law of interaction that he took to be in the form K_0/r^n, where K_0 is a constant and n is an integer. Evaluation of the desired rate ($\delta Q/\delta t$) required integration over ϕ and b such that he was able to obtain analytical results for the desired transport coefficients: shear viscosity (μ), thermal conductivity (κ), and mass diffusivity (D) by restricting attention to $n = 5$. (This restriction was needed to bring about a cancellation of the integrand's dependence on V.) Of the results obtained for these special "Maxwell molecules," in acoustics the following relationship between κ and μ is especially significant: $\kappa = F\mu c_v$, where c_v is the specific heat at constant volume and F is the dimensionless Maxwell-Eucken factor (using modern terminology). For gases having $\gamma = 5/3$, Maxwell's method correctly gives $F = 5/2$. (Because of an algebraic error first reported by Ludwig Boltzmann in 1872, this differs from Maxwell's claimed result in his Equation 149 where taking $\gamma = 5/3$ gives $F = 5/3$.) Acousticians today prefer to use the Prandtl number (Pr) in the modeling of thermoacoustic devices: $\mathrm{Pr} = \gamma/F$, which becomes 2/3 when $\gamma = 5/3$, in close agreement with modern measurements of the Prandtl number for monatomic gases.

As seen below, also relevant to subsequent developments in acoustics, Maxwell noted that his theory indicated a characteristic stress relaxation time of $\tau = \mu/P$. He estimated its value for air at normal conditions of 2×10^{-10} seconds, noting that "This time is exceedingly small, even when compared with the period of vibration of the most acute audible sounds; so that even in the theory of sound, we may consider the motion as steady during this very short time and use the equations we have already found, as has been done by Professor Stokes" (Maxwell, 1867, p. 83). In a broader context, his paper included the viscoelastic nature of gases.

Thermodynamics, Cosmogony, Molecules, Atoms, and Controversy

By 1868, Maxwell's attention was drawn to various issues of cosmogonic significance. For example, from thermodynamic considerations, he considered what he termed "physical indications of a beginning and an end" (Harman, 1995, p. 367). He examined related issues in his BAAS address of 1870 along with the stability of molecular processes as indicated by the similarity of terrestrial and stellar spectra, drawing attention to the wisdom of the perspective associated with the Christian faith in Hebrews 11:3 of the Biblical New Testament (Marston, 2016). (Here, Maxwell indicated " ... we seem to have advanced along the path of natural knowledge to one of those points at which we must accept the guidance of that faith by which we understand that 'that which is seen was not made of things which do appear'" [Maxwell, 1870, p. 421]). Better known, however, was his discussion of molecules having "the essential character of a manufactured article" in his BAAS address in September 1873 (Maxwell, 1873b) that drew criticism from John Tyndall in his widely publicized *Belfast Address* at the BAAS meeting in August 1874 (Tyndall, 1874). In January 1874, Maxwell had developed aspects of his own reasoning in an anonymous book review in *Nature* (Anonymous, 1874; Marston, 2007). In 1875, he expanded his observations and theistic perspective in an article on *Atoms* for the *Encyclopedia Britannica* by examining favorable "collocations" and "instances of benevolent design" (Niven, 1890). Many scientists even doubted the reality of atoms and molecules at that time. During this period, Maxwell also considered the consequences of a hypothetical superhuman "agent" or "doorkeeper" (eventually known as Maxwell's demon) that had the ability to sense molecular motion and utilize that information (Flood et al., 2014).

Maxwell, Stokes, Rayleigh, and Peer Review

In Britain during the Victorian era, the Royal Society of London published the most prestigious scientific journals: the *Proceedings* and the *Philosophical Transactions*. Typically, an abstract of scientific results would be read at a meeting of the Royal Society, followed by peer review of a full-length manuscript administered by the Society's secretary, a position held by George Gabriel Stokes (1819-1903) from 1854 to 1885. Often, the anonymity of reviewers was not maintained during the peer-review process. It was expected that opinions of the more senior reviewers would be respectfully valued during the review process, although there is some evidence that highly original manuscripts would be published even if the reviews were not in full agreement. (See, for example, the discussion of William Thomson's review of Maxwell's 1865 electromagnetic theory paper given in Marston [2016].) We know about Maxwell's growing involvement in the peer-review process in the 1860s and more specifically with papers associated with acoustics because of compilations of his reviews by Harman (1995, 2002). The first of this category of manuscript reviewed by Maxwell was from Stokes himself (Stokes, 1868), a paper still recognized as the first thorough analysis of sound production by bounded vibrating objects. Maxwell considered the paper "an important contribution to Mathematics and to Acoustics" and was favorable to its publication (Harman, 1995, p. 415). The next important example in this category is Maxwell's review of the manuscript by Rankine (1870), the first of the papers leading to the famed Rankine-Hugoniot relationships of shock wave physics. Rankine's manuscript emphasized the thermodynamics of waves and Maxwell was clearly supportive (Harman, 1995), incorporating Rankine's approach in the relevant section on wave propagation in his text *Theory of Heat* (Maxwell, 1871). In his associated discussion of the "condensation" of a sound wave becoming "more sudden" as the wave propagates, he illustrated the process with the "waves of the sea on coming into shallow water becoming steeper in front and more gently sloping behind, till at last they curl over on the shore" (Maxwell, 1871, Chapter 15).

The next examples concern Lord Rayleigh (birth name John William Strutt until mid-1873), who became a major contributor to acoustics. It is appropriate to first review early aspects of Strutt's life and education. Strutt entered Trinity College Cambridge in January 1861, and there are several parallels between his studies and those of Maxwell a decade earlier. It is possible that Strutt first met Maxwell when Maxwell was involved in the composition of the Cambridge Mathematical Tripos Examination in the late 1860s. They began to correspond on scientific and mathematical topics by May 1870 and interacted at the Liverpool BAAS meeting in September 1870. In December 1870, Maxwell wrote Strutt about the statistical nature of the second law of thermodynamics and his idea of a superhuman "doorkeeper." **Figure 3** shows Strutt's appearance at about that time. By May 1873, their relationship had progressed so that when Maxwell learned that Strutt was planning to write a book on acoustics, he wrote to him suggesting, "Why not call it *Theory of Sound*?" (Harman, 1995), advice that Strutt followed (Rayleigh, 1877). To appreciate Maxwell's early scientific influence on Strutt, notice that Strutt's first two publications (in 1869 and 1870) concern applications to electromagnetic phenomena of the analysis of field energy in Maxwell's 1865 dynamical theory (Rayleigh, 1964). In the second edition of his *Theory of Sound*, Rayleigh added several sections expanding on Maxwell's approach to "Electrical Vibrations." Another example of Maxwell's influence concerns Rayleigh's application in Section 226 of his book of Maxwell's graphical method of drawing field lines and equipotential curves to the construction of modal curves of vibrating plates. Maxwell, however, was also the beneficiary of early interactions with Rayleigh from his existing replies to Rayleigh's letters from 1870 to 1872 that included discussions of the construction of streamlines for fluid flow (partially analogous for electrical current flow) and Rayleigh's remarks on Maxwell's Lagrangian approach to the dynamics of circuits (Harman, 1995).

Other interactions concern Maxwell's reviews of Strutt's manuscripts (Harman, 1995, 2002). The first of these is his review of Strutt (1871) on the improved analysis of the frequency of gas-filled acoustic resonators through the approximation of (in modern terminology) the radiation reactance. Strutt needed to approximate the kinetic energy of oscillating flows of air close to the openings of air-filled resonators. To do this, he considered incompressible flow lacking friction and extended an analogy with electrical current flow within conductors similar to one explored by Maxwell in the mid-1850s. Strutt was able to place bounds on the analogous electrical conductance problem and hence on the resonance frequency of interest. In his review, Maxwell was favorably impressed by the method, suggesting some improvements and additional applications including the estimation of electrical capacitance. It appears that Maxwell retained a copy of his review because the electrical applications of Strutt's

Figure 3. *Rayleigh's self-portrait from the early 1870s scanned from a copy of Strutt (1924) owned by the present author.*

method that he developed in Articles 102 and 306 of his *Treatise* (with acknowledgments) closely parallel aspects of his review. Neither Maxwell nor Strutt may have noticed at this time some similarities to William Thomson's 1849 approach to related calculations of hydrodynamic kinetic energy (Harman, 1995). Maxwell also suggested in his review that Strutt consider the lowering of the natural frequency caused by the deficient rigidity of the container holding the oscillating volume of gas. Strutt included such a discussion in the publication (with acknowledgment to Maxwell), although he found the lowering to be negligible.

The aforementioned papers by Stokes, Rankine, and Strutt were reprinted in the ASA series *Benchmark Papers in Acoustics* in the 1970s and 1980s (Lindsay, 1973, 1974; Beyer, 1984). Maxwell's remaining reviews of Strutt's manuscripts concern items for the *London Mathematical Society* (Rayleigh, 1964; Harman, 1995, 2002). These include the 1873 paper introducing what is now known as the "Rayleigh dissipation function." Maxwell's review is important for its recognition of other applications (in modern terminology, the stability of systems with feedback and the nature of wave dispersion); he also suggested Strutt should consider reciprocity relationships, something that Strutt eventually pursued. In a lecture during the autumn term of 1873, Maxwell used Rayleigh's dissipation function, the notes from the lecture having been taken and retained by Charles Darwin's son, G. H. Darwin (Harman, 1995). Maxwell also reviewed Osborne Reynolds' paper concerning wind-induced refraction of sound for the *Proceedings of the Royal Society* (Harman, 2002).

Support for the Kinetic Theory from Acoustics of a Monatomic Gas, 1876

Up to the mid-1870s, it remained difficult to understand the measured values of $\gamma = c_p/c_v$ for any gas, the weakness of the kinetic theory noted by Maxwell at the end of his 1860 paper. Eventually, Kundt and Warburg (1876) used an acoustical measurement to determine γ for mercury vapor to be 1.666, a value consistent with the kinetic theory combined with a proper interpretation of the nature of the gas molecules.

It is first appropriate to explain the technique Kundt developed in the late 1860s for visualizing acoustic standing waves in gases confined in a glass tube. **Figure 4** shows a "Kundt tube" as depicted in a general textbook. A long solid rod is supported near its midpoint by a stopper (K-K). The rod is set into longitudinal vibrations by rubbing the rod, and the associated vibrations of the metal plate (*a*) at the end of the rod set up acoustic standing waves in the gas within the glass tube held in a horizontal position. Kundt found that when a light powder was present on the floor of the glass tube, the powder would move and accumulate in bands separated by a spacing of half a wavelength for the acoustic wave in the gas.

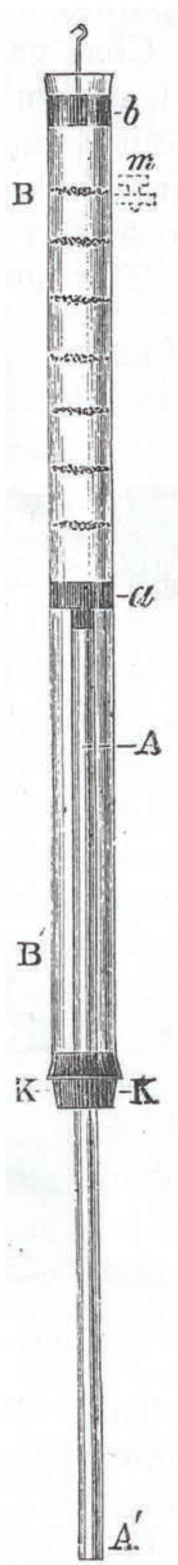

Figure 4. *Diagram of Kundt's method for measuring the wavelength of sound scanned from Atkinson (1890). Sound waves driven in the gas by the oscillating plate (a) are reflected from an adjustable stopper (b). The resulting standing acoustic wave in the gas causes powder to accumulate at regularly spaced intervals as depicted in this diagram.*

The experiment published in 1876 was more sophisticated due to the properties of mercury vapor. To motivate their measurements, Kundt and Warburg explained "there is currently an unresolved contradiction between experiment and theory" (in translation), indicating that it would be helpful to measure γ for a monatomic gas. Mercury vapor was proposed for investigation because it was interpreted as monatomic, provided hydrogen gas was viewed as diatomic. Either they or a colleague (Herr Baeyer, who suggested considering mercury vapor) was aware of the suggestion promoted by Cannizzaro in 1860 that molecular weights needed to be reinterpreted by assuming hydrogen gas to contain only diatomic molecules. Kundt and Warburg reduced the uncertainty in their measurement by comparing wavelengths measured at the same frequency for mercury vapor (at a known elevated temperature) with that of air at a known temperature. Consequently, their determination of γ for mercury relied on separate accurate results for the value of γ for air that had recently become available. In their application of kinetic theory, they interpreted mercury vapor as consisting of smooth spherical atoms not possessing rotational kinetic energy such that Maxwell's parameter in **Equation 2** becomes $\beta = 1$ so that they predicted $\gamma = 5/3$, in agreement with their measurements.

We can be confident of Maxwell's early appreciation of Kundt and Warburg's result because of a postscript he contributed to an obscure publication (Preston, 1877). It is first appropriate to introduce the author, S. Tolver Preston (1844-1917). One of Maxwell's London collaborators in electrical measurements was Fleeming Jenkin, also an assistant of William Thomson (later known as Lord Kelvin) in the engineering of long-distance telegraphy. Preston had assisted Thomson and Jenkin with telegraphy, although by the mid-1870s, he was in need of employment. Maxwell was favorably disposed toward Preston, indicating in a letter to P. G. Tait (another mutual friend) in December 1876, "[Preston] has really a good head if it were only trained a little and is no paradoxer" (Harman, 2002, p. 551). Preston had written Maxwell on December 5, 1876, evidently enclosing a manuscript of much of the publication (Preston, 1877). In his letter as well as in his publication, he endeavored to examine qualitative relationships between c, the speed of sound in gases, and the kinetic theory of gases (Garber et al., 1986). Preston's publication is important because of a second postscript, designated as P.S. (2), containing results "worked out mathematically" by Maxwell and an associated mention of the velocity of sound determination for "vapour of mercury" by Kundt and Warburg: Maxwell's stated result for that situation is that the speed of sound becomes $c = (5^{1/2}/3)v_{rms}$, where $v_{rms} = \sqrt{<v^2>}$ is the root-mean-square (rms) velocity from Maxwell (1860). His result was based on the assumption of "no movement of rotation" caused by molecular "encounters" (i.e., collisions) between the spherical molecules. Maxwell's result may be arrived at as follows. (1) The measurements of Kundt and Warburg were consistent with $\gamma = 5/3$, which from **Equation 2** requires $\beta = 1$, corresponding to no rotational kinetic energy. (2) In modern notation, Maxwell's (1860) result for the average translational kinetic energy per molecule is such that $(M/2)<v^2> = (3/2)k_B T$, where k_B is now known as Boltzmann's constant. (3) Also in modern notation, Boyle's law gives $(P/\rho) = k_B T/M$. These relationships combine to give Maxwell's asserted relationship between the speed of sound (c) and v_{rms}. Preston restated this relationship in a paper in *Nature* in 1878, adding that Maxwell's expression requires a slight additional correction in the case of most gases because of the movements of rotation developed at the collision of molecules. Maxwell noted in his 1879 analysis of thermal transpiration that Kundt and Warburg's result indicated that molecules of mercury gas "do not take up any sensible amount of energy in the form of internal motion" (Niven, 1890). His attention to their result is also implied in Campbell and Garnett's overview (1882, p. 569) of Maxwell's research on gases.

Preston's 1876 letter to Maxwell and his 1877 publication are also important because he explicitly mentions Waterston's (1858) qualitative attempt to relate the propagation of sound in gases with an earlier more qualitative approach to the kinetic theory of gases. This appears to be the only clear indication that Maxwell ever became aware of Waterston's prior interest in kinetic theory. It is noteworthy, however, that from the abstract of Maxwell's initial presentation for the 1859 BAAS, he planned to apply his kinetic theory of gases to the propagation of sound. That application was, however, not mentioned in his associated publication (Maxwell, 1860). It is plausible that because Maxwell couldn't resolve the aforementioned difference between the kinetic theory prediction for γ and the value implied by measured sound speeds in air (typically close to γ of 1.41), he neglected to pursue that application. In 1878, in one of his final papers, Maxwell gave a rigorous derivation of his energy equipartition theorem for gases independent of particular properties assumed of molecules (Niven, 1890).

After Maxwell: Applications and New Physics

In an early advance after Maxwell's death, Hendrik A. Lorentz (1853-1928) considered sound propagation in gases in 1880 by examining the near-equilibrium molecular distribution function (Kox, 1990). Lorentz obtained Laplace's result ($c = \sqrt{\gamma P/\rho}$) and introduced a new transport coefficient corresponding to the one eventually known as the bulk viscosity. Maxwell's proofs of the energy equipartition theorem remained controversial, although by 1900, Rayleigh strongly supported Maxwell's derivation of 1878. Given the difficulties in predicting the heat capacities of common gases, Rayleigh's support appears to have contributed to Kelvin's concerns in his famous 1901 address, "Nineteenth century clouds over the dynamical theory of heat and light" (Garber, 1978). Eventually, the incorporation of rotational and vibrational motion of diatomic gases (and other molecular gases) required quantum mechanical reasoning and the recognition of characteristic rotational and vibrational activation temperatures typically lying respectively far below and far above room temperature (Rushbrooke, 1949). In 1916-1917, Sidney Chapman and David Enskog extended the approach of Maxwell (1867) to different molecular force laws. Molecular velocity measurements with molecular beams in a vacuum in around 1930 eventually confirmed Maxwell's predicted velocity distribution. Herzfeld and Litovitz (1959) described the advances in the understanding of absorption and dispersion of sound in gases in the 1950s as a significant "generalization" of Maxwell's 1867 analysis of relaxation; the inclusion of timescales associated with the relaxation of rotational and vibrational excitations was also needed. By the 1970s, energy equipartition between the modes of complex macroscopic dynamical systems was found useful for "statistical energy analysis" in acoustics. In other developments, eventually $b\, db\, d\phi$ in **Equation 3** became known as a differential molecular scattering cross section, and by the 1930s, there was interest in their quantum mechanical evaluation using scattering phase shifts. Partially analogous expressions have been recently reintroduced in acoustics for the radiation force on spheres (Zhang and Marston, 2016). In Maxwell's January 1874 anonymous book review, he examined the consequences of an instantaneous reversal of the direction of motion of "every particle in the universe" so that "everything would run backwards." That thought experiment, commonly associated with an 1876 publication of Josef Loschmidt, was also considered in February 1874 by William

Thomson. A recent demonstration of time-reversed water waves uses a related concept (Bacot et al., 2016; Marston, 2017). To some extent, Maxwell's relationship with acoustical theory parallels his relationship with other areas of physics: it reflects the depth and breadth of his thought, interests, and responsibilities.

Biosketch

Philip L. Marston, pictured here with his wife Trude, studied physics at Seattle Pacific College and received a MS degree in electrical engineering and a PhD in physics from Stanford University. Following postdoctoral research with Robert E. Apfel at Yale University, he joined the faculty of Washington State University, Pullman. He received the Acoustical Society of America (ASA) Silver Medal in Physical Acoustics in 2003 and is a Fellow of the ASA and a Senior Member of the Optical Society of America.

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