The Tuning Fork: An Amazing Acoustics Apparatus

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It seems like such a simple device: a U-shaped piece of metal with a stem to hold it; a simple mechanical object that, when struck lightly, produces a single-frequency pure tone. And yet, this simple appearance is deceptive because a tuning fork exhibits several complicated vibroacoustic phenomena. A tuning fork vibrates with several symmetrical and asymmetrical flexural bending modes; it exhibits the nonlinear phenomenon of integer harmonics for large-amplitude displacements; and the stem oscillates at the octave of the fundamental frequency of the tines even though the tines have no octave component. A tuning fork radiates sound as a linear quadrupole source, with a distinct transition from a complicated near-field to a simpler far-field radiation pattern. This transition from near field to far field can be seen in the directivity patterns, time-averaged vector intensity, and the phase relationship between pressure and particle velocity. This article explores some of the amazing acoustics that this simple device can perform.

A Brief History of the Tuning Fork

The tuning fork was invented in 1711 by John Shore, the principal trumpeter for the royal court of England and a favorite of George Frederick Handel. Indeed, Handel wrote many of his more famous trumpet parts for Shore (Feldmann, 1997a). Unfortunately, Shore split his lip during a performance and was unable to continue performing on the trumpet afterward. So he turned his attention to his second instrument, the lute. Being unsatisfied with the pitch pipes commonly used to tune instruments at the time, Shore used his tuning fork (probably an adaptation of the two-pronged eating utensil) to tune his lute before performances, often quipping "I do not have about me a pitch-pipe, but I have what will do as well to tune by, a pitch-fork" (Miller, 1935; Bickerton and Barr, 1987).

It took more than a hundred years before Shore's tuning fork became an accepted scientific instrument, but starting in the mid-1800s and through the early 1900s, tuning forks and Helmholtz resonators were two of the most important items of equipment in an acoustics laboratory. In 1834, Johann Scheibler, a silk manufacturer without a scientific background, created a tonometer, a set of precisely tuned resonators (in this case tuning forks, although others used Helmholtz resonators) used to determine the frequency of another sound, essentially a mechanical frequency analyzer. Scheibler's tonometer consisted of 56 tuning forks, spanning the octave from A_3 220 Hz to A_4 440 Hz in steps of 4 Hz (Helmholtz, 1885, p. 441); he achieved this accuracy by modifying each fork until it produced exactly 4 beats per second with the preceding fork in the set. At the 1876 Philadelphia Centennial Exposition, Rudolph Koenig, the premier manufacturer of acoustics apparatus during the second half of the nineteenth century, displayed his Grand Tonometer with 692 precision tuning forks ranging from 16 to 4,096 Hz, equivalent to the frequency range of the piano (Pantalony, 2009). Keonig's Grand Tonometer was purchased in the 1880s by the United States Military Academy and currently resides in the collection of the Smithsonian National Museum of American History (Washington, DC; see tinyurl.com/keonig). For his own personal use, Koenig made a set of 154 forks ranging from 16 to 21,845.3 Hz; he achieved this decimal point precision at a frequency he couldn't hear by using the method of beats as well as the new optical method developed by Lissajous in 1857 (Greenslade, 1992). Lissajous' method of measuring frequencies involved the reflection of a narrow beam of light from mirrors attached to the tines of two massive tuning forks, oriented perpendicular to each other, resulting in the images that now bear his name (Guillemin, 1877, p. 196, Fig. 135 is one of the earliest images of Lissajous and his optical imaging tuning fork apparatus for creating these figures).

From the beginning, it was observed that touching the stem of the fork to a surface would transmit the vibration of the fork to the surface, causing it to vibrate. In the mid-1800s, Ernst Heinrich Weber and Heinrich Adolf Rinne

https://doi.org/10.1121/AT.2020.16.2.48

introduced tuning fork tests in which the stem of a vibrating tuning fork is touched to various places on a patient's skull to measure bone conduction; these tests have since become standard tools for the clinical assessment of hearing loss (Feldmann, 1997b,c). Similarly, in 1903, Rydel and Seiffer introduced a tuning fork with a graduated scale at the tines that is currently used to measure nerve conduction in the hands and feet (Martina et al., 1998).

From an educational viewpoint, the tuning fork has long been an important apparatus for demonstrations and experiments in undergraduate physics courses. Lincoln (2013) lists several tuning fork activities, including using an adjustable strobe light to see the tines vibrating; using a microphone and an oscilloscope to observe the frequencies of a fork and the variation in intensity as the fork is rotated; using a fork with a resonator box (Bogacz and Pedziwaitr, 2015) to demonstrate sympathetic resonance and beats; attaching a mirror to one of the tines to reproduce the original Lissajous figures; measuring the speed of sound with a fork and a cylindrical tube partly filled with water; and observing how the frequency of a fork depends on temperature.

Fork Frequencies, Tine Length, and Material Properties

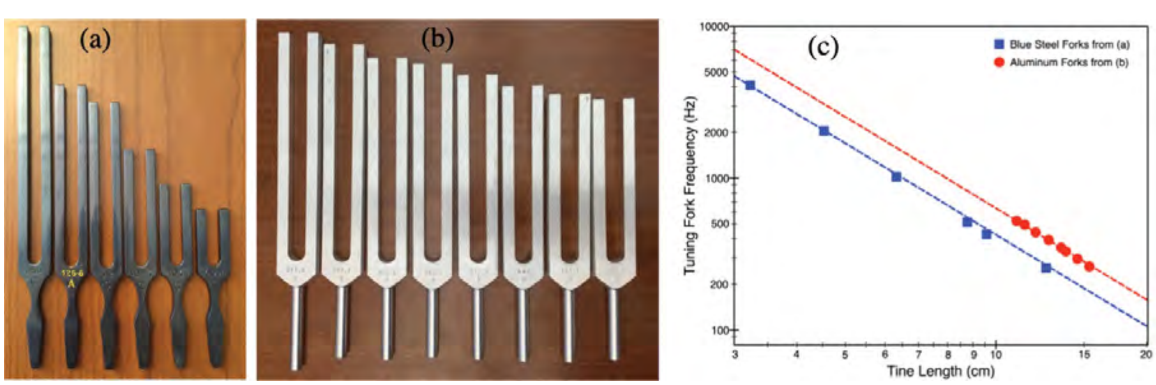
The frequency (*f*) of a tuning fork depends on its material properties and dimensions according to

$$f \propto \frac{A}{L^2}\sqrt{\frac{E}{\rho}} \qquad (1)$$

where L is the tine length, E and ρ are the Young's modulus of elasticity and density, respectively, and A is a factor determined by the cross-sectional shape and thickness of the tines (Rossing et al., 1992). For tuning forks made from the same material and having the same tine shape and thickness, the frequency depends on the inverse square of the tine length. **Figure 1**, **a** and **b**, shows a set of forks with frequency ratios of an octave starting at 256 Hz (with an extra fork at 426.6 Hz) and a set of tuning forks with frequencies corresponding to the notes of a musical scale starting at "middle" C_4 261.6 Hz, respectively. A plot of frequency versus tine length verifies that the frequency increases as the inverse square of the tine length (**Figure 1c**).

Since the mid-1800s when tuning forks began to be used as precision acoustical measurement devices, it has been known that the frequency of a tuning fork also depends strongly on temperature (Miller, 1926), with the frequency of a steel fork decreasing by 0.01% for every 1°C increase in temperature (Greenslade, 1992). In fact, some of the precision forks manufactured by Koenig in the late 1880s were stamped with the specific temperature at which the frequency would be accurate (Pantalony, 2009). Undergraduate student experiments report the frequency of a steel fork dropping by 1.0 Hz over a temperature increase of 55°C

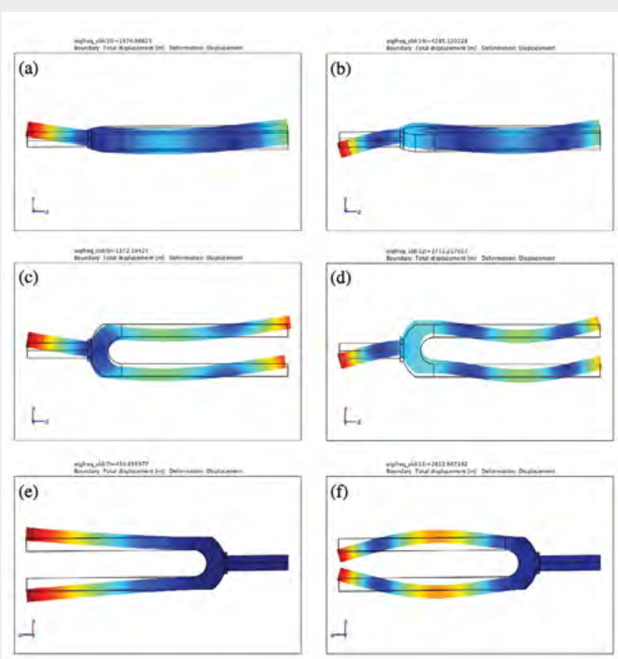
Figure 1. *Sets of tuning forks of the same material and tine cross-section dimensions but of different tine lengths.* ***a:*** *Set of blue steel forks with octave frequency ratios starting at 256 Hz, with an extra fork at 426.6 Hz.* ***b:*** *Set of aluminum forks forming a musical scale starting on "middle"* C_4 *261.6 Hz.* ***c:*** *Frequency versus tine length for the steel forks in* ***a*** *(****blue squares****) and the aluminum forks in* ***b*** *(****red circles****). Both dashed lines represent a power law of the form* $f \propto L^{-2}$.



(Bates et al., 1999) and a drop of 70 Hz for an aluminum fork as the temperature increased by 280°C (Blodgett, 2001).

The dependence of frequency on the properties of the material from which a tuning fork is made can be a useful means of giving students a tangible experience with the properties of various metals and other materials. Burleigh and Fuierer (2005) and Laughlin et al. (2008) manufactured different collections of 17 tuning forks with identical dimensions but made from a variety of metals, polymers, acrylics, and woods and used them with students in a materials course to explore how the frequency, duration, and amplitude of the tuning fork sound depends on material properties. The material from which a tuning fork is constructed is also important for noneducational applications; MacKechnie et al. (2013) found that a steel tuning fork was more likely to produce a negative test result than an aluminum fork when administering the Rinne test for clinical assessment of conductive hearing loss.

Figure 2. *Flexural bending modes for a tuning fork.* ***Red****, antinodes with maximum amplitude;* ***dark blue****, nodes with zero amplitude.* ***Top:*** *out-of-plane bending modes for a 430-Hz tuning fork.* ***a:*** *first bending mode at 1372 Hz.* ***b:*** *Second bending mode at 3,731 Hz.* ***Center:*** *in-plane bending modes for a 430-Hz tuning fork.* ***c:*** *First bending mode at 1,974 Hz.* ***d:*** *Second bending mode at 4,285 Hz.* ***Bottom:*** *symmetrical in-plane modes of a 430-Hz tuning fork.* ***e:*** *Fundamental mode at 430 Hz.* ***f:*** *"Clang" mode at 2,612 Hz.*



Flexural Bending Modes and Natural Frequencies

A tuning fork that is freely suspended (not held at the stem) will exhibit a number of flexural bending modes similar to those of a free-free bar. **Figure 2**, **a** and **b**, shows the first two out-of-plane flexural bending modes of a free tuning fork, and **Figure 2**, **c** and **d**, shows the first two in-plane bending modes. Because the fork does not have a uniform cross section along its length, the displacement amplitudes and the node positions are not symmetrical about the midpoint of the fork, something that is similar to the bending modes of a nonuniform baseball bat (Russell, 2017). However, these free-free mode shapes are not typically observed when the fork is held at the stem. Instead, the normally observed vibrational mode shapes, the shapes that give rise to the sound of the fork, are symmetrical modes in which the tines move in opposite directions (Rossing et al., 1992), as shown in **Figure 2**, **e** and **f**.

When vibrating in the fundamental mode, the tines of a handheld fork flex in opposite directions, like a cantilever beam. The second mode has a node roughly one-fourth of the tine length from the free end. An impact at this location will excite the fundamental but not the second mode; this is where to strike the fork to produce a pure tone. A fork should be impacted with a soft rubber mallet or struck against a relatively soft body part, like the knee or the pisiform bone at the base of the palm opposite the thumb (Watson, 2011). A fork should never be struck against a hard tabletop or hit with a metal object; doing so will excite other vibrational modes besides the fundamental and it could possibly dent the fork, changing its frequency.

The Frequencies of the Fundamental and the "Clang" Mode

When a tuning fork is struck softly, the resulting sound is a pure tone at the frequency of the fundamental symmetrical mode of the tines, as shown in **Figure 2e**. The spectrum in **Figure 3a** is for a soft impact on the tines of a 432-Hz tuning fork and shows a single, narrow peak at 432 Hz, 60 dB above the noise floor. **Figure 3b** shows that when this same 432-Hz fork is given a slightly harder impact at the tip of the tine, both the fundamental and also the second mode are excited. The second mode, called the "clang" mode, has a frequency of 2,605 Hz for this fork, which is slightly more than six times the frequency of the fundamental. The overtones of a tuning fork are not harmonics.

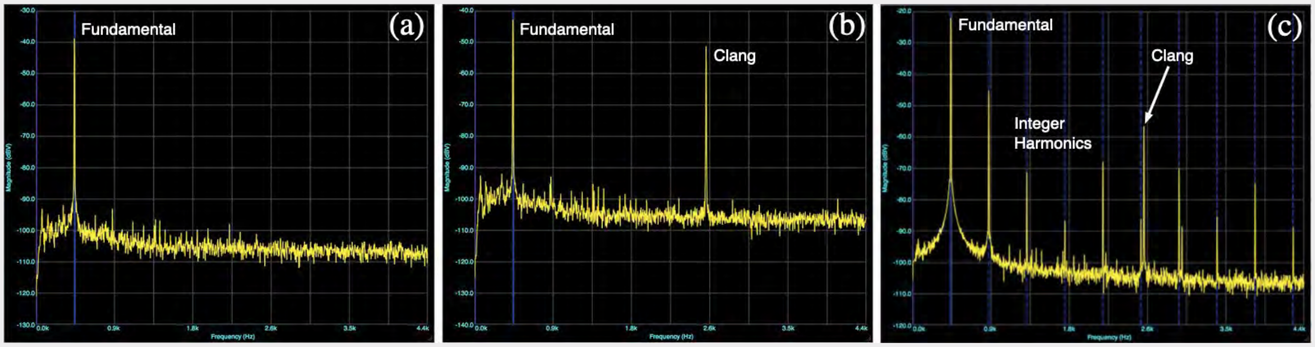


Figure 3. *Frequency spectra resulting from striking a fork: a soft blow (a); a harder blow at the tip of the tines (b); a very hard blow (c). See text for explanation.*

What boundary conditions would be appropriate for modeling the vibrational behavior of a tuning fork? Chladni (1802) approached the tuning fork by starting with a straight bar, free at both ends, and gradually bending it into a U-shape with a stem at the bottom of the U. The popular acoustics textbook by Kinsler et al. (2000, pp. 85-86) states that "The free-free bar may be used qualitatively to describe a tuning-fork. This is basically a U-shaped bar with a stem attached to the center." A different boundary condition was considered by Rayleigh (1894), who treated the tines of a tuning fork as being better modeled as a clamped-free bar. Who is correct? Well, a theoretical analysis of the boundary conditions for a beam undergoing flexural bending vibrations indicates that the frequency of the second mode of a free-free bar is 2.78 times the fundamental, whereas the frequency of the second mode of a fixed-free cantilever bar is 6.26 times the fundamental. The measured frequency of the clang mode, as shown in **Figure 3b**, suggests the clamped-free model is better.

The presence of the clang mode could pose problems for the clinical use of a tuning fork when assessing hearing health. Tuning forks with frequencies of 256 Hz and 512 Hz are frequently used for Rinne and Weber tests, and the corresponding clang modes near 1,600 Hz and 3,200 Hz, respectively, fall within the range of frequencies where human hearing is most sensitive. Thus care must be taken to strike the fork without exciting the clang mode to prevent misleading results during a clinical examination (Stevens and Pfannenstiel, 2015).

Nonlinear Generation of Integer Harmonics

When struck softly with a rubber mallet, a tuning fork produces a pure tone devoid of integer harmonics common to most musical instruments. However, an interesting result occurs when the fork is struck vigorously. If the tines are set into motion with a sufficiently large amplitude, the elastic restoring forces become nonlinear and the resulting radiated sound contains clearly audible integer multiples of the fundamental (Rossing et al., 1992). Helmholtz (1885, pp. 158-159) reportedly identified integer harmonics up to the sixth order for a large fork. The spectrum in **Figure 3c** shows the result of striking the fork hard enough to produce an audible "buzzing" and the amplitude of displacement at the end of the tines was visibly observed to be a couple of millimeters. This spectrum shows nine integer harmonics of the fundamental in addition to the clang tone.

Octave at the Stem

A more surprising observation is made when the stem of a vibrating fork is pressed against a sounding board or tabletop. The stem vibrates with a much smaller amplitude than the tines, but the tabletop is a much larger surface area so that the radiated sound, when a fork is touched to a surface, is considerably louder than the sound of the fork in air. Touching the stem to a surface produces an audible octave (exactly twice the fundamental frequency), even though the tines do not vibrate at the octave; the amplitude of the octave is often significantly louder than the fundamental (Rossing et al., 1992). A video demonstration of this phenomenon is found at y2u.be/NVUCf8mB1Wg.

The octave at the stem was noticed by Helmholtz (1855) and explored by Rayleigh (1899, 1912), who found that bending the fork tines inward could reduce the strength of the octave. However, an explanation of why only the octave and fundamental appear at the stem was not provided until much more recently. Boocock and Maunder (1969) developed a theoretical analysis, supported by experimental results, indicating that the presence of the octave at the stem is due to longitudinal inertia forces. They were able to explain Rayleigh's (1912) observation that bending the tines (thus offsetting the longitudinal imbalance) reduces the strength of the octave component. Sönnerlind (2018) developed a detailed computer model of a tuning fork and found that the octave motion in the stem is likely due to a nonlinear relationship between the vertical movement of the center of mass of the fork and the displacement of the tines. His model shows that a double frequency (octave) occurs because the center of mass of the fork reaches its minimum position twice per cycle, when the fork tines bend both inward and outward. Sönnerlind's model also indicates that the octave from the stem is more prominent for forks with longer tines and forks with tines having a square cross section (rather than a circular cross section).

The presence of the octave at the stem could affect the results of the Rinne and Weber hearing tests and the Rydel-Seiffer vibration sensitivity test because the stem is placed in contact with the skull, hands, or feet. This is why forks for assessing conduction hearing loss and nerve response to vibration are often fitted with weights at the tip of the tines because this reduces the presence of the octave at the stem.

The presence of an octave at the stem also has implications for piano tuners; touching a 440-Hz fork to the piano soundboard will produce a 440-Hz tone along with a stronger 880-Hz octave, and the 880-Hz octave from the tuning fork stem will beat with the A 880-Hz piano string that is tuned slightly sharp due to the intrinsic inharmonicity in piano strings. This very problem was posed as a question to me during my graduate school days, and answering the question was the beginning of my fascination with the acoustics of tuning forks (Rossing et al., 1992).

Directivity Patterns, Quadrupole Sources, and Intensity Maps

When a tuning fork vibrates in its fundamental mode, the tines oscillate in opposite directions, with each tine acting as a dipole source such that the two oppositely phased dipoles combine to form a linear quadrupole source (Rossing et al., 1992). The linear quadrupole is an interesting sound source because the sound field at near and far distances from the source exhibits distinct differences in directivity patterns, vector intensity maps, and the phase between pressure and particle velocity.

Quadrupole and Dipole Directivity Patterns

The nature of the quadrupole radiation may be demonstrated by rotating a tuning fork about its long axis while holding it close to the ear or near to the opening of a quarter-wavelength resonator tuned to the fork fundamental (Helmholtz, 1885, p. 161). During one complete rotation, there will be four positions where the resulting sound is loud, alternating with four regions where the sound is very quiet; the sound will be loud when the tines are in-line with the ear and also when the tines are perpendicular. However, if the fork is held at arm's length from the ear and rotated, only two loud regions will be heard, when the tines are in-line with the ear, and the previously loud regions when the tines are perpendicular to the ear will now be quiet. This variation in the loudness means that care must be taken regarding the orientation of the tuning fork tines with respect to the external auditory canal during the air conduction portion of the Rinne test (Butskiy et al., 2016).

Figure 4, a-c, compares the measured directivity patterns at increasing distances from a 426-Hz tuning fork with theoretical predictions for a linear quadrupole source. Measured sound pressure levels around a 426-Hz tuning fork vibrating in its fundamental mode agree very nicely with theory at all distances (Russell, 2000; Froehle and Persson, 2014). These data explain why one hears four loud regions when a fork is rotated close to the ear but only two loud regions when the fork is rotated at arm's length. It also explains why, if you listen very carefully, the sound is noticeably louder (about 5 dB) when the tines are aligned with the ear compared with when they are perpendicular.

If a fork is rigidly clamped at the stem, it may be forced into several other natural modes of vibration that radiate sound as a dipole source or as a lateral quadrupole source. **Figure 4, d-f**, shows measured directivity patterns for a 426-Hz tuning fork that was clamped at the stem and driven at an in-plane dipole mode at 257 Hz, an out-of-plane dipole mode at 344 Hz, and a lateral quadrupole mode at 483 Hz. The measured data agree well with the theoretical predictions for dipole and lateral quadrupole sources (Russell, 2000).

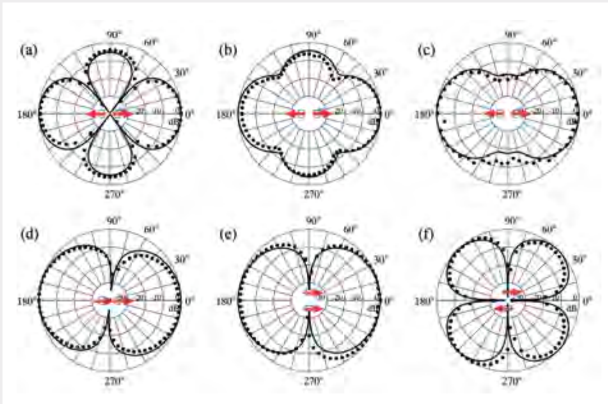


Figure 4. *Sound pressure level directivity patterns around a tuning fork.* ***Solid circles****, measurements;* ***solid curves****, theory for a linear quadrupole.* ***Red arrows****, relative direction of tine motion.* ***Top:*** *data for a 426-Hz fork vibrating in its fundamental mode at distances of 5 cm (****a****), 20 cm (****b****), and 80 cm (****c****).* ***Bottom:*** *data for the same 426-Hz fork driven into vibration as an in-plane dipole source at 275 Hz (****d****), an out-of-plane dipole source at 344 Hz (****e****), and a lateral quadrupole source at 483 Hz (****f****). Adapted from Russell, 2000, with permission.*

Acoustic Intensity Maps and Energy Flow Around a Fork

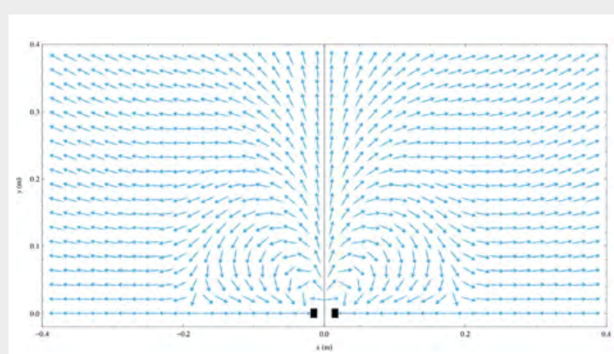
The transition from near-field to far-field radiation for a linear quadrupole source may be explored further by looking at the time-averaged vector intensity. The time-averaged acoustic intensity represents the net energy flow; it is a vector quantity with both magnitude and direction. In the far field from a simple source, the vector intensity points radially away from the source, indicating that the source is producing waves that carry energy away from the source in a roughly omnidirectional manner. However, the near field of a source may consist of regions where the energy swirls around, with no net outward flow.

Figure 5 shows a theoretical prediction of the time-averaged acoustic intensity vectors in two quadrants of the horizontal plane surrounding a tuning fork, modeled as a linear quadrupole source. The amplitude of the intensity for a quadrupole source falls off as the inverse of the fourth power of distance, so the direction of flow has been normalized to unit length to make it visible and to emphasize the directional property of the intensity. In the far field, the direction of the intensity vectors indicate that sound energy is propagating radially outward, away from the fork. The near field shows a much more interesting feature. Perpendicular to the fork tines (the vertical axis of the plot), energy is radiated away from the fork at all distances. But, in the direction parallel to the tines (the horizontal axis of the plot), energy is actually directed inward toward the fork in the near field. At a distance approximately 0.225 times the wavelength, the intensity drops to zero before changing direction and pointing outward for farther distances (Sillitto, 1966). Although the acoustic intensity vanishes at this location, the pressure does not drop to zero, and sound will still be heard without any change in loudness at this location. This theoretical prediction of the "swirling" of the energy in the near field of the fork has been experimentally verified through measurements of the time-averaged vector intensity using a two-microphone intensity probe; the measured data confirm the theoretical predictions (Russell et al., 2013).

Phase Relationship Between Pressure and Particle Velocity

An additional aspect of the transition between the near field and far field around a tuning fork is the relationship of the phase between pressure and particle velocity. In the far field of a spherically symmetrical source, the pressure and particle velocity are in phase with each other; both reach maximum and minimum values at the same time. In the near field, however, the pressure and particle velocity are 90° out of phase; when one quantity is at a maximum or minimum, the other quantity is zero

Figure 5. *Time-averaged acoustic intensity vectors in two quadrants around a tuning fork. The* ***two black rectangles in the center*** *represent the tines and the* ***arrows*** *indicate the direction of flow of acoustic energy. See text for explanation. Adapted from Russell et al., 2013, with permission.*



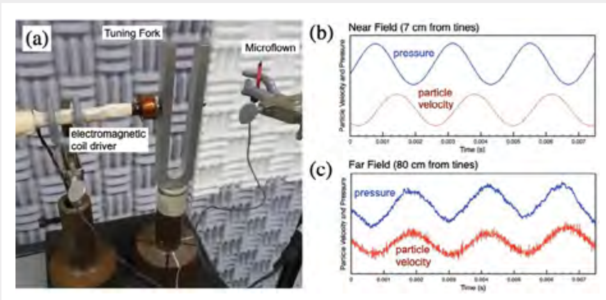


Figure 6. *A 426-Hz fork driven in its fundamental mode with an electromagnetic coil and a Microflown transducer (**a**) measures pressure and particle velocity in the near field at 7 cm from the fork (**b**) and in the far field at 80 cm from the fork (**c**). See text for explanation. Adapted from Russell et al., 2013, with permission.*

Figure 7. *The Acoustical Society of America Gold Medal is the highest award of the Acoustical Society of America. The medal shows a tuning fork vibrating with a large displacement amplitude and radiating sound waves. Photo courtesy of Elaine Moran, used with permission.*

and the quantities are said to be in quadrature. **Figure 6** shows measurements of the pressure and particle velocities made with a matchstick-sized Microflown transducer near a large 426-Hz fork. In the near field, at a distance of 7 cm from the tines, the pressure and particle velocity are seen to be nearly 90° out of phase to each other. But at a larger distance of 80 cm, in the far field, the pressure and particle velocity are nearly in phase. The quadrature phase relationship between pressure and particle velocity is a topic discussed frequently in upper level undergraduate and graduate acoustics courses covering spherical waves. However, even though I had known this for many years, as both a student and a teacher, the first time I obtained the experimental data in **Figure 6** was an exciting moment.

The Tuning Fork on the Gold Medal of the Acoustical Society of America

The humble tuning fork is a simple mechanical device that is capable of demonstrating a wide variety of complex vibroacoustic phenomena. Perhaps it is no surprise that this marvelous acoustical apparatus is prominently featured on the Acoustical Society of America (ASA) Gold Medal (**Figure 7**), the most prestigious recognition awarded by the ASA. It is interesting to note that the fork depicted on the medal appears to be vibrating with a sufficiently large amplitude so as to produce nonlinearly generated integer harmonics. However, whereas the shape of the fork looks similar to those made by Koenig, the radiated wave fronts are far too close together (relative to the fork dimensions); this fork must have a fundamental frequency much higher than the 21,845-Hz fork in Koenig's personal collection.

Acknowledgments

I thank my friend and former MS thesis advisor, Thomas D. Rossing, for introducing me to the fascinating acoustics of tuning forks back when I was his student at Northern Illinois University, DeKalb.

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