Sound in the World
Throughout human history, people and cultures have created sound for more than simple communication. For example, early humans likely made music using primitive flutes (Atema, 2014) and considered sound integral in the design of cities (e.g., Kolar, 2018). Furthermore, the Mayans designed structures at the ruins at Chichen Itza in Mexico that used sound for worship (Declercq et. al., 2004). Specifically, clapping in front of the stairs of the El Castillo pyramid creates a sound resembling a highly revered bird by way of a series of reflections up the stairs (available at bit.ly/3jPfOTk).

In addition to an interest in making sound, sound and vibration have also been thoroughly investigated by either empirical methods or philosophical arguments since as far back as Pythagoras (550 BCE), who applied his discoveries in mathematics to the harmonic ratios in music. He discovered that stringed instruments could be tuned, using small integer ratios of string length, so that they would consistently produce layered consonant musical intervals.

The interest and desire to study our acoustic environment continues to this day, but the methods we use have changed dramatically, and continue to change as new technologies emerge. Beginning in the seventeenth century with Robert Boyle, empirical investigation showed that sound is a vibration of conceptualized fluid particles transmitting energy from one place to another. Theoretical and empirical investigations are essential but more often require additional help to solve the problems at hand. Indeed, applying sophisticated computational methods, the basis of this article, provides a valuable tool in understanding and analyzing acoustics phenomena.

The Need for Computational Acoustics
The need for computational acoustics shows itself in the difficulty in most real-world physical investigations in acoustics, often requiring solving the acoustic wave equation. Indeed, there is potential for advancement in new areas of research not contained in the traditional areas by employing computational acoustics. This is already seen from the great developments and advancements in all areas of acoustics over several decades where the complexity has required extensive use of numerical methods, optimization, computational modeling, and simulation.

The relationships of computational physics to mathematics and computer science, the relationship between acoustics, mathematics, and computer science define computational acoustics as described by the Venn-type diagram shown in Figure 1.

The Wave Equation Explained
The wave equation enables the expression of motion in a wave, and it shows itself in every area of physics including acoustics, electromagnetism, quantum mechanics, and optics, to name a few. The equation provides the

Figure 1. Venn diagram showing the concept relationship of computational acoustics, indicating how it connects traditional acoustics with mathematics and computer science.
mathematical relationship between the variables of interest in acoustics, often the acoustic pressure or particle velocity and the speed of a wave. The equation relates the temporal and spatial changes to these variables, including dependence on the wavelength and frequency of the wave. As a consequence, the equation is a second-order partial differential equation of pressure or particle velocity and is three-dimensional. The pressure or particle velocity is dependent on three spatial directions and time.

In all areas of physics, solutions to problems involving the wave equation require specifying additional boundary conditions that depend on the geometry of the problem. Only in specific ideal cases with simple conditions and geometry are analytical solutions even possible. However, the wave equation is a powerful and useful tool for investigating the physics involved.

For most real problems of interest, the geometry involved is much too complicated to solve by any other means than by computational methods. For example, if one wanted to simulate the propagation of sound through the ear canal (Puria, 2020), the geometric structure would not be simple and defining real boundary conditions would make the problem too complicated to be solved any other way than by numerical solution of the wave equation.

Propagation of acoustic waves in a variety of environments is well understood and documented, but any real environment is overly complex and prediction of sound fields becomes impossible to solve analytically. For example, one may wish to determine the acoustic pressure field in a large area underwater in the ocean (e.g., Duda et al., 2019) where the environment, boundary conditions and spatial distributions of fluid properties are complicated. To solve a wave equation with such complexity, the problem is reduced to numerical solutions.

There are a bevy of techniques discussed in this article for solution of the wave equation in various situations. Several of these techniques are numerical methods applied to solve the equations directly without approximations, whereas others require a successive approximation of results.

The Emergence of Computational Methods
Since its invention in the 1930s, the digital computer has been used to solve difficult problems in physics. Early uses were in areas of nuclear physics where they performed simulations on ballistics and particle evolution for the development of the atomic bomb.

Monte Carlo Simulation
Several techniques and algorithms were developed at the Los Alamos (NM) National Laboratory by Jon von Neumann as part of his work on the atomic bomb, leading to what we now know as Monte Carlo simulations. As one might expect, Monte Carlo applications involve any phenomena that could be modeled as random or spontaneous, such as playing games of chance at a casino. Some phenomena that are modeled this way include radioactive decay and the random nature of thermal motion (Landau and Price, 1997). Additionally, Monte Carlo simulations can be used to model sound propagation in the atmosphere (Burkatovskaya et al., 2016) where multiple scattering and the turbulent nature of the atmosphere (Blanc-Benon et al., 2002) can be taken into consideration.

Continued work in computational physics led to the discovery of chaotic behavior in nonlinear dynamics where deterministic mechanical systems exhibited seemingly random states of motion. The theoretical underpinnings of mechanics had existed for nearly half a century before computer technology made it possible to make the complicated computations needed to simulate the interactions.

Early Use of Computers in Acoustics
An early mention of using computers in acoustics is provided by a talk given at the 62nd meeting of the Acoustical Society of America by Schroeder (1961) on novel uses in room acoustics. In his abstract, Schroeder spoke of using digital computers to simulate complicated transmission of sound in rooms and simulation of spatial and frequency responses in rooms using Monte Carlo techniques. Schroeder’s insight revolutionized architectural acoustics. Computational methods have proven enormously powerful in predicting acoustic performance of interior spaces and have enhanced the ability of the specialist to design spaces acoustically, such as in concert halls (Sviolja and Xiang, 2020).

The decades of improvement in computer technology and computational performance have allowed greater use of such numerical methods for acoustic wave propagation, scattering, radiation, and other acoustically related phenomena. This, in turn, has enhanced discovery and problem solving. Simulations of different phenomena have provided
ways to investigate interactions that previously were unapproachable due to the complex nature of acoustics.

Computational acoustics, which is a combination of mathematical modeling and numerical solution algorithms, has recently emerged as a subdiscipline of acoustics. The use of approximation techniques to calculate acoustic fields with computer-based models and simulations allows for previously unapproachable problems to be solved.

The increasing computational nature of acoustics, especially in all the traditional areas, has provided a cross-disciplinary opportunity. The purpose of this paper is to show an overview of the various techniques used in computational acoustics over several of the traditional areas. I am more familiar with applications in underwater acoustics and physical acoustics, but many of the same techniques used in those areas can be applied in other areas (see Table 1 for articles in *Acoustics Today* that discuss the use of similar techniques).

In addition, applications of machine learning (ML) that are being used in artificial intelligence research and areas of data science are also being exploited to advance research into areas including acoustic oceanography, engineering acoustics, and signal processing. This is by no means an exhaustive list, but it brings a familiarization to the areas and applications of computational acoustics and the methods found therein.

### Modern Computational Methods

The numerical methods of computational acoustics are focused on taking the continuous equations and differential equations from calculus and turning them into linear algebraic equations, which are amenable to solution on digital computers. In the case of a concert hall with complex geometries that are not open to an analytic solution, computational acoustics would enable an acoustics engineer to compute a numerical solution to the wave equation to help the engineering design process, as discussed recently by Savioja and Xiang (2020).

Two of the more popular methods are the finite-difference method (FDM) and finite-element method (FEM). The FDM is a class of numerical techniques related to a general class of numerical methods known as Galerkin methods (Jensen et al., 2011; Wang et. al., 2019) that treat derivatives as algebraic differences and the continuous function in question, such as the sound field, is calculated at various points of space (Botteldooren, 1994).

For example, Figure 2 shows how to break up the space with a grid where the sound field is calculated at an individual element in space. Each point is calculated through iteration via a computational algorithm. The calculations are often simple enough that they could be performed with pencil and paper or a basic calculator. However, if the procedure needs to be applied to many points, there may need to be thousands to millions of computations, thereby requiring a digital computer.

In contrast to the FDM, the FEM is another numerical technique used for calculating sound fields based on dividing up a space or structure into individual elements, each of which is assumed to be constant. The space/structure is broken into individual elements, each calculated through iteration via a computational algorithm. The calculations are often simple enough that they could be performed with pencil and paper or a basic calculator. However, if the procedure needs to be applied to many points, there may need to be thousands to millions of computations, thereby requiring a digital computer.

### Table 1. Some relevant articles published in *Acoustics Today*

<table>
<thead>
<tr>
<th>Authors</th>
<th>Topic</th>
</tr>
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<tbody>
<tr>
<td>Ahrens et. al., 2014</td>
<td>Sound field synthesis</td>
</tr>
<tr>
<td>Bruce, 2017</td>
<td>Speech intelligibility, signal processing</td>
</tr>
<tr>
<td>Bunting et. al., 2020</td>
<td>Computational acoustics</td>
</tr>
<tr>
<td>Burnett, 2015</td>
<td>Computer simulation of scattering</td>
</tr>
<tr>
<td>Candy, 2008</td>
<td>Signal processing, model-based machine learning beginnings</td>
</tr>
<tr>
<td>Duda, et. al., 2019</td>
<td>Ocean acoustics</td>
</tr>
<tr>
<td>Greenberg, 2018</td>
<td>Deep learning, languages</td>
</tr>
<tr>
<td>Hambri and Fahnline, 2007</td>
<td>Structural acoustics, modeling methods</td>
</tr>
<tr>
<td>Hawley et. al., 2020</td>
<td>Musical acoustics</td>
</tr>
<tr>
<td>Puria, 2020</td>
<td>Bioacoustics, hearing</td>
</tr>
<tr>
<td>Stone and Shadle, 2016</td>
<td>Speech production, modeling, computational fluid dynamics</td>
</tr>
<tr>
<td>Treeby, 2019</td>
<td>Biomedical acoustics</td>
</tr>
<tr>
<td>Vorländner, 2020</td>
<td>Virtual reality and music</td>
</tr>
<tr>
<td>Wage, 2018</td>
<td>Array signal processing and localization</td>
</tr>
<tr>
<td>Wilson et. al., 2015</td>
<td>Atmospheric acoustic propagation</td>
</tr>
<tr>
<td>Zurk, 2018</td>
<td>Underwater acoustic sensing</td>
</tr>
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</table>

These papers have either a computational focus or computational relationship.
up into a mesh, which looks like a wire grid applied to the structure of various shapes, often triangles. The points of the chosen mesh shape are called nodes, and these define the shape of the mesh. The goal of the method is to sum the contribution of each element to the sound field. **Figure 3** shows the conceptualization of dividing up a structure with a simple grid using a triangular mesh instead of the square boxes in **Figure 2** that divide up a structure. Although the method seems complicated, the main idea is simple.

For real-life problems, the FDM and FEM are not exclusive, and they are often applied at the same time on modern high-performance computing platforms. The FDM is simple in its application but requires some initial knowledge of conditions. The FEM is more adaptable and accurate but often requires more input data to apply.

**Direct Numerical Simulation**

The complete mathematical treatment of complex acoustic problems in fluids begins with a set of partial differential equations known as the compressible Navier-Stokes equations. These equations describe both the flow of the fluid and the aerodynamically/hydrodynamically generated sound field. These equations are statements of conservation of momentum and mass in the fluid, describing all the dynamics.

Due to this coupling of fluid dynamics and acoustics, both fluid variables and acoustic variables may be solved directly by rewriting the equations into a form that can be fully simulated via a computer program or software package such as COMSOL or ANSYS. These types of packages are good at performing simulations of systems where multiple kinds of physics are involved, like a problem involving sound transmission through living tissue where there could be heating, density variations, and fluids in motion. Often what is required is a very precise numerical resolution due to the large changes in the length of the scales between acoustic and flow variables due to fluids in motion. The use of direct numerical simulation is often computationally challenging and is unfitting for most applications without the use of high-performance computing.

Although direct numerical simulation may be a limitation, it is often the first approach to use on a variety of problems. One such application is calculating the compressional and shear speeds of elastic waves in a material of interest utilizing measured backscattered acoustic data from a sphere made of the material. The compressional and shear speeds are related to the scattered sound in a complicated way but can be determined for spherical objects. I am not going into the complex mathematics behind the calculations; however, the method is to compute the theoretical backscattering function (Faran, 1951; Chu and Eastland, 2014). This function has discontinuities, called nulls, that are related to the compressional and shear speeds of sound in the material. The null locations and separations are dictated by these speeds.

Beginning with an initial guess of the speeds, the backscatter form function is determined. Backscattering data from the target are then matched to the form function by relating the error in the null locations and separations. Based on the selection of arbitrary nulls in the data using any nonlinear least squares method (e.g., Levenberg-Marquardt), an
optimization process is selected. The goal is to minimize the error in the fitting of the data by iteratively updating the form function. The initial guess of the speeds is updated and used to recompute the form function until the desired level of error in the cost function is achieved. As one can imagine, this is a brute force method and can be computationally demanding. However, it is effective. An example of the data output is shown in Figure 4.

The simulation proved to be accurate in the predicted values to within 6% on the shear speed and less than 5% on the compressional speed with only 22 iterations. The computational time using MATLAB on a personal computer took nearly 10 minutes. If a higher precision is desired, the minimum error can be adjusted to get more iterations but will take much longer.

The first of these various numerical methods often used to determine the sound pressure in a computational acoustics problem is the FDM. The method takes the continuous differential equation that describes the phenomena and breaks it into a finite algebraic set of equations. The details are left out in this article for brevity; however, the usefulness of this method is hard to deny. The method is used by breaking up the space into a grid of points, and the sound field is calculated with small changes in the field based on the nearest grid points in the space. The solution is found by solving numerically using small steps in space and time. This technique is used in multiple areas and works well for wave propagation and scattering problems.

By way of an illustration (see Bunting et. al., 2020), the application of the wave equation and discretization shows the power of computational acoustics. Assuming harmonic time dependence of pressure and applying the wave equation, one obtains the Helmholtz equation. The Helmholtz equation describes steady-state wave propagation in physics and relates to acoustic wave propagation through either the particle velocity or pressure in a fluid.

There are multiple methods utilizing a known result of the acoustic wave equation to compute the acoustic field of a sound source. A general solution for wave propagation can be written as an integral over all present sources, which are summarized as integral methods. The origin of the acoustic source must be determined a priori from some other method (e.g., a FEM simulation of a mechanical structure). The integral is taken over all sources relative to the time of the source of the signal. The sound wave arrives later at a given receiving position. Common to all integral methods is that changes in the speed of sound between the source

Figure 4. Dashed black curve, theoretical target strength determined from the form function; blue solid curve, backscattered data-determined target strength of a 64-mm copper sphere, comparing theory and experiment; red circles, the three chosen nulls to be matched to the data, determining the compressional and shear sound speeds in the material to within at least 6% accuracy. Work was done on sonar calibration for biomass estimated acoustic surveys by the author at Northwest Fisheries Science Center (Seattle, WA).

Figure 5. a: Array of acoustic point sources arranged as several hexagonal distribution where cross-range is the lateral left/right dimension and elevation is the up/down vertical dimension as a demonstration of the method. b: An acoustic color plot of the simulated acoustic sound pressure level measured at a location 10 m from a source array of 31 point sources being driven in unison at 10 kHz. Yellow is louder than darker blue to a level in decibels relative to 1 μPa of acoustic pressure.
and receiver positions cannot be justified by utilizing the theoretical solution of the wave equation.

An example application of an integral method is to calculate the acoustic field from a hexagonal array of sources treated as point sources. Figure 5a shows an example of an array of 31 sources arranged as a grouping of hexagons. The locations are determined computationally, with each node being treated as an acoustic point source. The field is summed over all sources, and the level is calculated at a given range. This could be done over time to create a movie of the acoustic field that can provide insight into how the acoustic wave propagates. The simulation is assumed to be underwater with a source frequency of 10 kHz. An example might be a source array, but the problem is easily simulated using integral methods. The array used in Figure 5a has the output given as a color plot in Figure 5b, respectively.

**Kirchhoff Integral**

Kirchhoff and Helmholtz were able to show that sound radiating from a localized source in a limited area can be described by enclosing this source area by an arbitrarily envisioned surface. The sound field inside or outside the chosen surface is calculated using the Helmholtz equation. The solution can be determined by the sum of a set of “basis” functions related to the geometry of the problem that can be used. The difficulty in the problem is determining the functions that work and is not described here due to being out of scope of this article. The calculated field on the surface directly follows from the wave equation.

A variation of the scheme allows one to calculate the pressure on the arbitrary surface using the normal particle velocity, which is the mechanism involved in acoustic transmission. The particle velocity perpendicular to the surface could be given by a FEM simulation of a moving structure. However, the modification of the method to avoid utilizing the acoustic pressure directly on the surface leads to snags, with enclosed volumes being driven at their resonant frequencies. This is a major issue in the implementation of the technique. To get around this limitation, the sound pressure is determined on the surface of the object first and then imaginary sources are added on its surface to cancel the normal particle velocity on the surface of the object.

An instance of the use of the Kirchhoff integral is to divide the physical domain into a smaller simpler set of parts for a more complex problem, which introduces the application of the FEM (Everstine and Henderson, 1990). This is another example of integral methods, but it solves the field by direct integration over the surface. The goal is to split the computational area into different regions so that the central acoustic equations can be solved with different sets of equations and numerical techniques.

For instance, simulating an idealized Helmholtz resonator (such as a violin or guitar) as a flower vase and solving the wave equation with boundary conditions becomes difficult due to the odd shape of the boundary. To solve this, therefore, the boundary is broken up into smaller pieces, and the acoustic field is calculated for each individual piece of the boundary. A concept figure showing what the boundary would look like is shown in Figure 6.

The method would then employ breaking up the vase into physical elements, as in Figure 3, where all the corners of the element are broken into nodes. The method just sums the acoustic field from each individual element for each node in space, assuming some constant coefficient given as \( \bar{p}_I \) for each element, approximated from the boundary and field equations. Each element would be given as some shape function given as \( N_f^e \), which was a triangle in Figure 3. The total acoustic field is determined as a sum of each individual contribution such as \( p(x,y,z) = \sum_{i=1}^{N_f^e} \bar{p}_i \cdot N_f^e \), where \( x, y, \) and \( z \) are the obligatory spatial variables.

**Machine Learning and Other Contributions**

Several significant contributions have been made in different areas of investigation with the applications of computational acoustics. One of these is the incorporation of machine learning techniques to improve the accuracy and efficiency of acoustic simulations.
of acoustic simulation methods into virtual reality systems (Vorlander, 2013, 2020). These types of systems can have real-time performance due to the advances in technology and have become paramount in the entertainment industry. Additionally, virtual and augmented realities have been employed in training and as a diagnostic tool. In the past, there used to be latency or slowing down of simulations due to the huge amounts of data being generated. However, this is not as significant a problem anymore given advances in computer technology. As a result, sound synthesis and production of indoor/outdoor surroundings can be combined with three-dimensional stereoscopic display systems through data fusion (e.g., Vorlander, 2020). The research and design applications have led to improved reality for video games and similar systems. The user experience is enhanced by adding accurately synthesized sound and allowing the listener to be able to move unrestrictedly, e.g., turn the head, to be able to perceive a more natural situation. Moreover, the improved synthesis algorithms (e.g., Gao et al., 2020) can be used to provide more realistic conditions for psychoacoustic tests. Sound synthesis algorithms based on deterministic-stochastic signal decomposition have been applied to synthesize pitch and time scale modifications of the stochastic or random component of internal combustion engine noise (Jagla et al., 2012). The method uses a pitch-synchronous overlap-and-add algorithm, used in speech synthesis, that exploits the use of recorded engine noise data and the fact that the method does not require specific knowledge of the engine frequency. The data-based method used for speech synthesis, noise analysis, and synthesis of engine noise just mentioned is similar to what is used in ML. Applications of ML seem to have no limits in the data-driven world of today.

ML methods are based on statistics and are excellent at detecting patterns in large datasets. Applications in acoustics are fertile ground for research into ML for things such as voice recognition, source identification, and bioacoustics (e.g., Bianco et al, 2019). With technologies like Alexa or Google Home, voice recognition investigations are needed to allow the technology to work with people having different accents or pronunciations or speaking different languages. The algorithms must utilize huge datasets of recorded voices to teach the computer system to “learn” based on input. Models are developed of voices pronouncing certain common words used for searching. Variations are compared statistically to the model where the model can be improved based on additional inputs of data. The computer algorithm from the system using it essentially “learns” and incorporates that knowledge into its dataset. Although much of the research into ML and techniques are done in areas of computer science, the applications of the methods into acoustics have driven some of the more recent advances. A major method of ML, called deep learning, based on artificial neural networks that work through several layers, train systems to do everything from synthesizing music to being able to perform better than the human ear for recognition (Hawley et al., 2020).

Summary and Conclusions

The large variety of methods and applications outlined here is hardly an exhaustive depiction of computational acoustics. Due to limitations in my knowledge and the space and time to do so, only a brief introduction to the field could be given. However, hopefully, I was able to make the case for the need for the field of computational acoustics and the variety of areas of application. The uses of computational methods have driven discovery and improved understanding in a variety of areas of acoustics including sound synthesis, voice recognition, modeling of acoustic propagation, and source identification. Several techniques have been used to aid in the design of new automotive technologies by modeling the mechanical interactions of structures with different moving parts and the fluids involved. Several of these methods are not only being used in engineering acoustics, but they are also being employed for space design for concert halls and classrooms. This type of modeling has improved noise suppression in a variety of mechanical systems. Computational techniques are being used in modeling and simulation in signal processing to utilize ML methods in the investigation of acoustic source identification and classification. The methods are being applied to areas of animal bioacoustics to aid in species identification for population monitoring, avoiding direct interaction with the animals. The methods and applications of computational acoustics are only going to grow over years to come and have become a fruitful and rewarding area of research.

Disclaimer

The opinions and assertions contained herein are my private opinions and are not to be construed as official or reflecting the views of the United States Department of Defense, specifically, the US Navy or any of its component commands.
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